

MISALLOCATION AND PRODUCTIVITY: MICRO EVIDENCE FROM BANGLADESH

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ABSTRACT

An important determinant of aggregate measured productivity is how resources are allocated across heterogeneous production units. Idiosyncratic distortions from institutional policies and factors can be a source for resource misallocation resulting in lower total factor productivity (TFP) and aggregate output. Distortions are like taxes and subsidies that create heterogeneity in production units—they cause before-tax marginal revenue products to be higher in production units that face disincentives (taxes), and to be lower in production units that receive incentives (subsidies). In the absence of distortions, production units equate marginal products with their corresponding factor prices, making resource allocations efficient because the more productive units proportionately use more resources. In the presence of distortions, they are equated with both factor prices and distortions, making equilibrium allocations dependent on both individual TFPs and distortions and resulting in aggregate output and TFP losses.

In Chapter 2, I develop a measure of aggregate agricultural TFP using a model of perfect competition where production is subject to decreasing returns to scale. Using detailed household farm-level data from Bangladesh agriculture, I measure the observed gross TFP. I find that it is not land but capital and intermediate inputs that are misallocated the most in Bangladesh. If resources were hypothetically reallocated across farms according to their marginal products, then aggregate TFP could increase by more than 120% relative to the observed TFP.

In Chapter 3, I develop a model to measure industry-level and aggregate TFP of Bangladesh manufacturing, with and without distortions. Each narrowly defined industry is perfectly competitive but production is carried out by heterogeneous firms. I use firm-level data from the manufacturing industries of Bangladesh to measure dispersions in the marginal revenue products of capital and

labor. I find that if allocations were efficient, then aggregate TFP could increase by as much 95%. Of the two factors, capital is more misallocated than labor in most of the industries.

In Chapter 4, I explore the quantitative implications of common versus sector-specific misallocation. I develop a two-sector model of agriculture and non-agriculture, each with an endogenous distribution of production units. In each sector the distribution of active production units depends on the productivity of the unit operation and idiosyncratic distortions that the unit faces in that sector. I capture idiosyncratic distortions as a producer-sector-specific output tax that stands in as a catch all for the policies and institutions that alter the relative prices faced by producers within each sector. I use micro-level data on manufacturing plants and farms and a quantitative framework in order to measure the distortions. I calibrate my model to the micro data from Bangladesh with observed distortions and conduct a set of counterfactual experiments. Overall I find that while eliminating distortions in each sector would raise productivity in that sector and at the aggregate level it is only improvements in agricultural labor productivity that generate substantial structural change in the economy.

DEDICATIONS

To my daughters, *Anaya* and *Siara*.

To my mother, *Fatema*, who has always loved me; and my later father, *Abdul Jalil*, who had always believed in me.

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1 Chapter One

Misallocation and Productivity in the Literature

One important question in development economics is why some countries are so rich and others are so poor. Two stylized facts capture important differences between rich and poor countries. First, there are large differences in output per worker between rich and poor countries (Restuccia et al., 2008).¹ Second, there are large cross-country differences in aggregate TFP. While differences in physical and human capital per worker can explain a big part of the income differences, a large unexplained part captured by TFP—the dominant source—remains. Productivity is strongly correlated with GDP per worker; therefore, understanding differences in TFP is a key step in understanding cross-country income differences (Klenow and Rodriguez-Clare, 1997; Prescott, 1998; Hall and Jones, 1999; Hsieh and Klenow, 2008).

One underlying cause of low TFP in poor countries is the inefficient use of existing technologies or slow diffusion (adoption) of productive technologies from advanced to poor countries, both of which lower firm-level productivities (Howitt, 2000; Klenow and Rodriguez-Clare, 2005).² These are models of within-firm inefficiency, however, and compare similar production units across countries. The focus in the recent literature on productivity is on many production units within an industry or sector; aggregate TFP depends on both firm-level TFPs and how factor inputs are allocated across these production units. If the market operates efficiently, then only the surviving profit maximizing production units receive inputs according to their individual TFPs. On the contrary, if a market has production units that should not operate or that receive inputs disproportionately

¹They estimate, using 1985 data, that aggregate output per worker in the richest 5% of the countries was 34 times larger than that of the poorest 5%.

²Differences in cross-country TFPs are also explained by blocking of better technologies by vested group (Parente and Prescott, 2000).

to individual TFPs, then there is misallocation of factor inputs, as a result of which the economy has lower aggregate TFP. In other words, misallocation deprives production units within sectors of exploiting their true potentials in terms of productivity.

There are two approaches in the literature, a direct and an indirect approach, that address misallocation of factors as an important source of measured aggregate or sectoral TFP differences across countries. In both approaches, there is a departure from the representative production unit paradigm and units are heterogeneous. The direct approach studies the effects of specific policies, trade barriers, institutional factors and market imperfections along one or more dimensions that can be empirically measured on the allocation of resources within an economy or sector. For instance, Hopenhayn and Rogerson (1993) and Lagos (2006) show that labor related policies, such firing taxes, unemployment insurance and employment protection, distort labor allocation and generate TFP losses. Also, Guner et al. (2008) show that countries like India, Japan and Italy have restrictions on firm sizes that lead to loss in aggregate output.³ Trade barriers affect productivity of both manufacturing and agriculture: For instance, Waugh (2010) shows that dispersion of manufacturing productivity increases with trade barriers, and Tombe (2012) shows that trade barriers are responsible for low agricultural productivity and lack of trade in food in poor countries. Epifani and Gancia (2011) show that trade barriers have heterogeneous effects on mark-ups, misallocating resources.

In many countries high costs of doing business (such as entry costs, regulations, heavy taxation and financial frictions) encourage informal entrepreneurship, which is small and unproductive. In these countries, the coexistence of informal and formal establishments is a source of misallocation. Leal (2010) and D’Erasmus and Moscoso Boedo (2012) provide evidence of informality and low

³As another country example, low labor productivity in Brazil is due to labor market regulations (McKinsey Global Institute, 1998.)

productivity associated with high costs of doing business. Credit market imperfections or constraints have two effects: one, they discourage really productive entrepreneurs to the point that they can never become active; two, they misallocate capital by limiting the amount of capital that existing production units have access to. There are many works that estimate the effects of various types of capital market imperfections on TFP. Notably, Banerjee and Duflo (2005) provide microeconomic evidence on misallocation of capital due to credit constraints and institutional failures leading to cross-country TFP differences. Caselli and Gennaioli (2013) and Buera and Shin (2010) model misallocation of capital to managerial talent.

Specific policies or institutional factors that are sources of misallocation are difficult to measure, let alone be compared across countries. For example, in countries where corruption is rampant and a norm, if producers are politically connected then they can enjoy many benefits, such as, low interest rate (from state-owned financial institutions), special tax breaks, subsidies or protection against competition. Consider a manufacturing plant: whereas borrowing should be up to the point where its marginal product of capital is equal to its cost (interest rate), if politically connected, it may end up borrowing a lot of capital, probably depriving other potentially more efficient plants. In short, it's difficult to directly measure the effects on two different plants that enjoy different level of political connection. As another example, consider what happens when transportation and communication networks are poor. A farm close to the wholesale market is better informed about prices than a farm that is located far from the market. These are common characteristics in developing countries that create heterogeneity in production units, with significant consequences on output and productivity.

In order to capture the effects of policies and institutions that create heterogeneity, without reference to specific policies, an indirect approach is taken where it is assumed there are plant or

farm-specific wedges (labelled as distortionary taxes) in the marginal products of all factors: capital, land, labor and intermediate inputs, causing production units to produce amounts different from what would be dictated by their productivity and factors. Essentially, misallocation happens because marginal products are not equated across producers for each factor. In a seminal paper, Hsieh and Klenow (2009) estimate wedges in marginal products of labor and capital across plants within narrowly-defined industries in China and India; moreover they show that, if these gaps were hypothetically reduced to the extent observed in the U.S., then there would be significant gains in manufacturing TFP. In an earlier work, Banerjee and Duflo (2005) suggest that resource misallocation due to the gaps in the marginal products of capital plays an important role in understanding India's low manufacturing TFP relative to that of the US. So, rather than looking at specific policies (the direct approach), I can look at the distribution of individual wedges and estimate the extent of misallocation affecting TFP and output. Whereas policies are hard to measure, wedges can be extracted from the first order conditions of a production unit's optimization problem (Chari et al., 2007). This indirect approach in the literature has given rise to new models (of production units within industries or sectors) that are very important to understanding measured TFP differences across countries.

In my thesis I do the following: First, I separately study the extent of misallocation in the two important sectors of Bangladesh: agriculture and manufacturing. I used detailed micro-level data on production units in manufacturing and agriculture for Bangladesh to investigate the extent of resource misallocation and evaluate the implied productivity losses.⁴ Bangladesh is one of the most densely populated countries in the world, making its agriculture vital for employment and staple food production. Equally important is its manufacturing, which has grown rapidly

⁴Until now, microdata from middle income countries were used for productivity experiments.

since the 1990s. However, the country is riddled with corruption, distortionary policies and poor institutions that create heterogeneity across all production units in all sectors. My contribution to the expanding literature on productivity and misallocation is that I follow the indirect approach to providing empirical evidence on sectoral TFP losses for a low-income country.⁵ Second, I estimate the distribution of idiosyncratic taxes (the underlying wedges), and then separately estimate the potential gain of resource reallocation in manufacturing and agricultural TFP of Bangladesh.

In the last chapter, I develop a two-sector model of agriculture and manufacturing featuring heterogeneous production units in each sector. I allow for production unit - level distortions in each sector. The model generates an endogenous distribution of farms in agriculture and an endogenous distribution of establishments in manufacturing, which are driven by the joint distribution of productivity and distortions in each sector. I use micro-level data from Bangladesh to discipline the model. In particular, I use farm-level data from agricultural sector and establishment-level data from the manufacturing sector in Bangladesh to calibrate the joint productivity-distortion distributions in each sector. The micro data allow us to: (i) construct TFP for each farm and each establishment in my dataset, (ii) back out the idiosyncratic distortions they face, and (iii) identify empirically the moments of the joint distribution of productivity and distortions, including their correlation. I match a number of other aggregate and sectoral statistics for Bangladesh so that the model economy constitutes an equilibrium. Equipped with my calibrated model I then conduct a set of counterfactual experiments where I “shut down” in turn: (1) agriculture-specific idiosyncratic distortions; (2) manufacturing-specific idiosyncratic distortions; and (3) all distortions across both sectors. These experiments allow me to quantitatively assess the role that each type of distortion plays for struc-

⁵See Bergoeing et al.(2002), Galindo et al. (2007), Alfaro et al. (2008), and Bartelsman, et al. (2008) for related empirical evidence in other countries.

tural change (captured by the share of employment in agriculture), average production unit size, agricultural labor productivity, manufacturing labor productivity, and development (aggregate labor productivity). I find that while eliminating distortions in each sector raises productivity in that sector and at the aggregate level, it is only improvements in agricultural productivity that generate structural change in the economy, with a large reallocation of labor from lower productivity agriculture to higher productivity manufacturing. Overall I find that eliminating all distortions could reduce the share of employment in agriculture by 12 percentage points and increase output per worker by about 40 percent.

2 Chapter Two

Misallocation in Agriculture: Firm-level Data from Bangladesh

2.1 Introduction

To what extent does misallocation exist in agriculture? One important fact is that if aggregate output is separated into agriculture and non-agriculture, then there is a 78-fold difference in agricultural output per worker between the richest and poorest 5% of countries (Restuccia et al., 2008).⁶ There are similar findings in other works, such as in Caselli (2005) estimating that there is a 45-fold difference in agriculture output per worker between the richest and poorest 10% of countries. These differences are much larger than aggregate productivity differences. Poorest countries also have a much higher share of labor in agriculture relative to rich countries; 86% of total employment in the poorest 5% of all countries is in agriculture whereas it is only 4% in the richest 5% (Restuccia et al., 2008). Since there are large cross country differences in labor productivity, agriculture must account for large share of the difference in aggregate TFP.⁷

Adamopoulos and Restuccia (2014a) show that a three quarters of cross-country productivity differences can be explained by farm-level policies (inheritance rules, progressive taxes, subsidies, land reforms and tenancy restrictions) that misallocate resources from large-productive to small less-productive farms in poor countries. Using micro data for the Philippines, Adamopoulos and Restuccia (2014b) show that land reform had large negative impact on its agricultural productivity. More recently, using micro data for Malawi, Restuccia and Santaaulàlia-Llopis (2015) show that if land were efficiently used, then its productivity could increase by 3.6 fold. While Adamopoulos and

⁶These statistics are from Restuccia et al. (2008) measures: GDP from PWT 5.6 and agriculture data are from the Food and Agricultural Organization of the United Nations.

⁷Differences are large relative to non-agriculture.

Restuccia (2014) and Restuccia and Santaaulàlia-Llopis (2015) trace misallocation to land policies, I ask whether land is the primary source of misallocation in other low income countries. To investigate the extent of misallocation and its effect on output and productivity, I consider Bangladesh agriculture, and follow the indirect approach which requires that I measure farm-specific distortions.

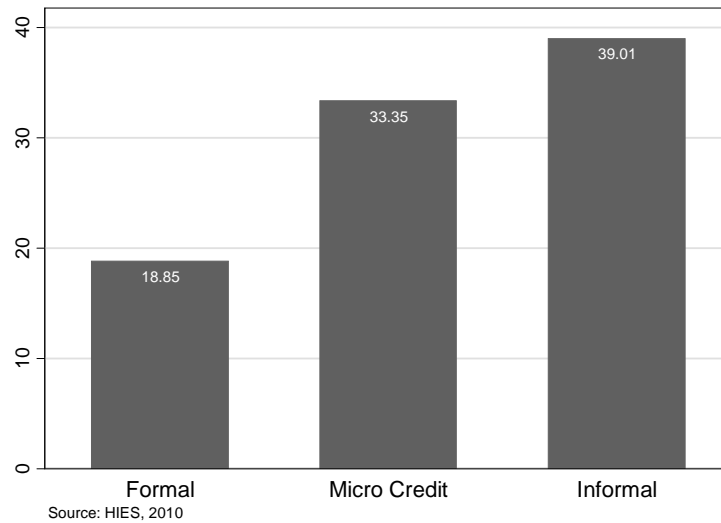
There are a few works in which idiosyncratic distortions are embedded into the optimization problems of farms. Notably, Restuccia and Rogerson (2008) show that firms that face heterogeneous factor prices, due to firm-specific output distortion, generate lower output and TFP. Hsieh and Klenow (2009) work with output and capital distortions (as residuals from firm's optimization problem) and show that if the extent of misallocation were equated to the US level one, then manufacturing TFP in China would increase between 30% and 50%, and in India between 40% and 60%. In this chapter, I build a static model in the spirit of Hsieh and Klenow (2009), with the following differences: First, while they focus on manufacturing firms, I focus on agricultural farms. Second, they use the Melitz (2003) framework of monopolistic competition with CES preferences and constant returns to scale technology, whereas I use the Lucas Span-of-Control framework of perfect competition with decreasing returns-to-scale technology.⁸ Third, I estimate the TFP of gross output in order to capture the effects of distortions in intermediate inputs. Fourth, I work with distortions in capital, land, labor and intermediate inputs, whereas they work with only output and capital distortions. Similar to their methodology, I estimate the distortions that farms face from the residuals in the marginal value products of capital, land, labor and intermediate inputs. Then I estimate the potential gain in TFP by removing distortions and hypothetically reallocating resources across all agricultural farms corresponding to the optimal allocation. I find that if distortions were removed,

⁸The two models are essentially isomorphic; in this framework diminishing returns to production play the same role as diminishing returns to utility.

agricultural TFP could increase by more than 120%. More important, it is not land, as emphasized in the work of Adamopoulos and Restuccia (2014b), but capital and intermediate inputs that are misallocated the most in Bangladesh agriculture.

Capital distortions are pervasive in Bangladesh agriculture, and they manifest through the operations of many types of formal and informal financial institutions. Borrowing cost from the informal financial sector is significantly larger than from the formal financial sector.⁹ According to the Household Income and Expenditure Survey of 2010, while agricultural households paid a weighted interest rate of about 19% to formal financial institutions, the rates were much higher against borrowings from micro-credit and informal institutions (Figure 2.1). At farm-level, the effective interest rate was as high as 720% (HEIS, 2010).

Figure 2.1: Interest Rate



This chapter proceeds as follows. In section 2.2, I present some facts about Bangladesh that may serve to distort the allocation of factors across farms. In section 2.3, I develop a baseline model

⁹Friends, relatives, local money lenders, etc. comprise the informal financial sector.

of perfect competition with heterogeneous farms and derive a measure of aggregate TFP. In section 2.4, I describe in detail the dataset and how the idiosyncratic distortions are measured. In section 2.5, I provide evidence on misallocation based on the model developed in Section 2.3, and estimate TFP gains. Section 2.6 shows a few robustness results, followed by the conclusion in section 2.7.

2.2 Motivation

Bangladesh, an economy of nearly 160 million people, is still agriculture based as agriculture contributes 20% of GDP and employs nearly 31% of the labor force (aged 15 years and above). Except for Nepal and Afghanistan, in 2010, agricultural productivity (2nd column, Table 2.1) of Bangladesh was significantly lower than in other countries in South Asia.¹⁰ In the ranking of Ease of Doing Business (World Bank, 2015) Bangladesh ranks 177 out of 189, meaning that the regulatory environment is not very conducive to the starting and operation of a business. India, Pakistan and Sri Lanka rank much higher and their productivity was higher.

¹⁰Agricultural productivity here is defined as agriculture value added per worker, which measures the output of the agricultural sector (ISIC divisions 1-5) less the value of intermediate inputs. This includes forestry, hunting, and fishing as well as cultivation of crops and livestock production.

Table 2.1: Agricultural Labour Productivity and Ease of Doing Business

	Agricultural (2010) Output/Worker	Ease of Doing Business (2015)
Afghanistan	401	177
Bangladesh	519	174
Bhutan	634	71
India	666	130
Maldives	2901	128
Nepal	261	99
Pakistan	1048	138
Sri Lanka	935	107

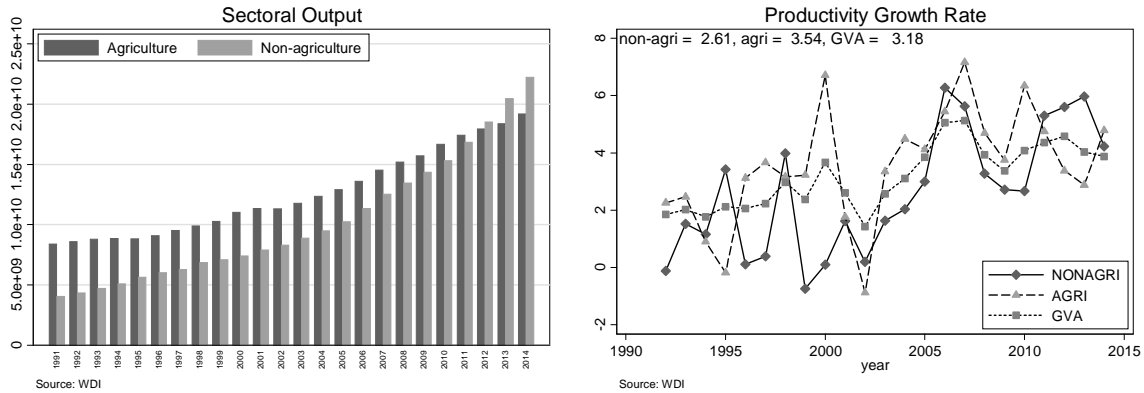
Column 2: Constant 2005 US\$, Source: WDI

Column 3: Doing Business (Index), Source: WBG

Over the last 25 years, while the real output of non-agriculture has increased by a factor of 5.5 fold, the real output of agriculture has increased by a factor of 2.3-fold (Figure 2.2). Furthermore, in terms of sectoral productivity, it is evident that the average productivity growth rate in agriculture was lower than in non-agriculture.¹¹

¹¹Sectoral productivity is measured as gross value output (constant 2005 US\$) divided by sectoral workers (labor force between the ages 15 to 64).

Figure 2.2: Output and Growth Rate of Agriculture



The question is, to what extent is agricultural resource misallocation responsible for low productivity and output in Bangladesh? There are many factors and policies, including market structure, of Bangladesh agriculture that potentially distort the allocation of resources across farms. An extensive survey of 1837 agricultural household by Barkat et al. (2010) provides insight into the market structure and factors that have heterogeneous effects on farmers.

The survey categorizes farmers into five groups: landless (owning less than 0.5 acres of land), marginal (owning between 0.5 and 1.49 acres), small (owning between 1.5 and 2.49 acres), medium (owning between 2.50 and 7.49) and large (owning above 7.5 acres). It is found that 25% landless farm household members have no education, followed by marginal (22%), small (15%) and medium farm household members (14.9%). For 83% of large farmers farming is not the primary occupation, whereas, for 92 landless and 93% marginal farmers farming is the primary occupation. The survey also finds that it is intimidating for Illiterate farmers to engage in formal institutions for credit.

Agricultural credit is vital for production (mostly to cultivate 3 types of rice variety: *Boro*, *Aman*

and *Aus*). According to the survey, 38% of farmers collected credit for agricultural activities. 58.2% used credit to buy fertilizer; 37.6% to pay wages; 27.1% to buy seeds. 8.1% of farmers used credit for power tiller and 12.% for a tractor. Credit was taken from formal (NGOs, 28%; *Krishi* Bank, 17%; government banks, 5.6%; private banks, 2.6%) and informal (relatives and neighbors, 30.4%; moneylenders, 6.6%; local *shomiti*, 5%; influential rich persons, 2.4%) sources. The study finds that 56.6% of landless farmers collected agricultural loans from informal sectors; for the marginal and small farmers it was 45.4% and 45.3% respectively. On the other hand, 66% of medium and 83.3% of large farmers accessed agricultural loans from formal sectors. This provides strong evidence that access to formal borrowing is increasing in farm size.

The official procedure to borrow from formal financial institutions is complicated and intimidating, particularly for farmers with low education. Poor farmers think it is easier to take credit from friends, relatives and neighbors (at very high effective interest rates). In rural areas, influential people discourage and misguide (even harass) poor farmers to engage in the formal channel. Besides, the formal private banking sector is not interested in serving poor (landless, marginal and small) farmers reflected by limited operations (and branches) of different banks in rural areas compared to NGOs (that provide microcredit) and informal cooperatives.

In rural Bangladesh the private sector supplies seeds and fertilizers (by 15,000 suppliers), but every single seller is chosen by the government, so, invariably, there are irregularities and corruption in the existing fertilizer distribution system. There have been numerous fertilizer crises over the last 5 decades, mostly due to policies.¹² Across all categories, the percentage of farmers who have experienced fertilizer deficit is decreasing in the land size. What's evident is that higher land-holding gives social power and influence, making it easier for larger farmers to avail fertilizer

¹²There were protracted fertilizer crises in 1974, 1984 and 1995.

more easily than others. Complaints reported by farmers are numerous, such as, price variations across different geographical areas; subsidy diverted to unintended beneficiaries; fertilizers sold above government approved prices; inadequate supply of fertilizers by quotas allocated to dealers in various districts; interference with the distribution mechanism by well-connected people, political heavyweights, and personnel entrusted with fertilizer sales; same fertilizer type for all farmers irrespective of crop grown and soil type; inaccurate assessment of the demand for fertilizer; adulterated fertilizer; smuggling and black marketing. The fertilizer distribution system has changed so many times and so frequently that there is confusion, knowledge and information gap between farmers and distributors. Instability in fertilizer distribution not only has manifested in price volatility, but also affected the landless, marginal and small farmers, who spent most of their borrowed capital on fertilizer. Although there is an official price for every type of fertilizer, not all farmers pay the same price for it. Fertilizers are not always available at convenient locations, and farmers have to buy them from distant places incurring significant transportation cost. Per unit transportation cost, therefore, for small farmers is high. Even after discounting skewed transportation cost, landless farmers paid 12.77 *taka* per kilogram for (solid) urea fertilizer on an average, whereas large farmers paid 12.43 *taka* per kilogram for the same variety. The difference, though perceptively small, is important for small farmers. The high price of fertilizer and financial constraints affect landless farmers the most and large farmers the least. Larger farmers inability to meet fertilizer requirement is primarily an issue of availability in times of need.

Finally, Bangladesh is so vulnerable to recurring floods, cyclones and droughts that it is ranked as the world's 5th most exposed country to natural disasters, all of which adversely affect output (Beck et al., 2012). Add to that, due to erosion and rising sea-level, the country loses low-lying

arable lands to the rivers and the Bay of Bengal.¹³ The consequences of natural calamities have varying effects on farms. While small farmers can become completely landless overnight, other farmers may become relatively bigger even after losing lands.¹⁴ Some regions of Bangladesh (particularly low-lying arable lands along river banks and the sea coast) are vulnerable to rising sea levels where farmers bear high costs of protecting lands against erosion (Karim and Mimura, 2008). Despite this high vulnerability, there are no formal commercial-sector crop insurance schemes available for the farmers (Akter et al, 2009). Enforcement of property rights is limited to those who are affluent and politically influential. It is not uncommon to see people with muscle usurp the lands of marginal and small farmers. There are traditional geography-specific norms that still exist to deal with middlemen in markets, credit collection and workers and in the dealings of land.

Having discussed the underlying idiosyncratic factors that may affect farmers' allocation of resources, in section 2.3 I build my model.

2.3 Technology

A production unit in agriculture is a farm, managed by a farm operator. Farm operators are heterogeneous in their productivity in managing their farms. Each farm operator, denoted by i , has access to a nested decreasing returns to scale technology, and uses capital (k), land (l), labor (n) and intermediate inputs (x) to produce a set of crops given by

$$y_i = s_i^{1-\gamma} \left[\left(k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta} \right)^{1-\mu} x_i^\mu \right]^\gamma \quad (1)$$

¹³1% of arable land is lost to accomodate the growing population, industrial activities (Islam and Hassan, 2013). Farmers sometimes lose their agricultural land for economic, social or political reasons.

¹⁴Many farmers migrate to cities after becoming landless or remain in rural areas and continue to work as hired workers for other farmers.

where $s_i^{1-\gamma}$ is the production efficiency (TFP) of the farm operator. The parameter $0 < \gamma < 1$ (referred to as the Lucas "span-of-control") governs the degree of decreasing returns to scale at farm level.¹⁵ $\alpha(1 - \mu)\gamma$ and $\mu\gamma$ are the share of capital and intermediate inputs respectively. In addition to heterogeneity in production efficiency (s_i), an operator faces four distinct idiosyncratic distortions. Separately, these are: capital distortion (τ_i^k) that affects the marginal product of capital; land distortion (τ_i^l) that affects the marginal product of land; labor distortion (τ_i^n) that affects the marginal product of labor; and intermediate inputs distortion (τ_i^x) that affects the marginal product of intermediate inputs. Each operator's profit is given by

$$\begin{aligned}\pi_i &= py_i - (1 + \tau_i^k)rk_i - (1 + \tau_i^l)ql_i - (1 + \tau_i^n)wn_i - (1 + \tau_i^x)zx_i \\ &= ps_i^{1-\gamma} \left[\left(k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta} \right)^{1-\mu} x_i^\mu \right]^\gamma - (1 + \tau_i^k)rk_i - (1 + \tau_i^l)ql_i - (1 + \tau_i^n)wn_i - (1 + \tau_i^x)zx_i,\end{aligned}\tag{2}$$

where py_i is the gross output of all crops at weighted prices. The real factor prices of capital, land, labor and intermediate inputs are r, q, w and z respectively but altered by $(1 + \tau_i^k), (1 + \tau_i^l), (1 + \tau_i^n)$ and $(1 + \tau_i^x)$. Since distortions are farm-specific, the effective factor prices vary across farms. From the first order conditions of a farmer's optimization problem (2) the allocation of capital, land, labor and intermediate inputs are proportional to

¹⁵It reflects manager's span control over a farm.

$$\begin{aligned}
k_i &\propto s_i \left\{ \left[\frac{\alpha}{r(1+\tau_i^k)} \right]^{\alpha(1-\mu)\gamma+1-\gamma} \left[\frac{\beta}{q(1+\tau_i^l)} \right]^{\beta(1-\mu)\gamma} \left[\frac{1-\alpha-\beta}{w(1+\tau_i^n)} \right]^{(1-\mu)\gamma(1-\alpha-\beta)} \left[\frac{1}{z(1+\tau_i^x)} \right]^{\mu\gamma} \right\}^{\frac{1}{1-\gamma}}, \\
l_i &\propto s_i \left\{ \left[\frac{\alpha}{r(1+\tau_i^k)} \right]^{\alpha(1-\mu)\gamma} \left[\frac{\beta}{q(1+\tau_i^l)} \right]^{\beta(1-\mu)\gamma+1-\gamma} \left[\frac{1-\alpha-\beta}{w(1+\tau_i^n)} \right]^{(1-\mu)\gamma(1-\alpha-\beta)} \left[\frac{1}{z(1+\tau_i^x)} \right]^{\mu\gamma} \right\}^{\frac{1}{1-\gamma}}, \\
n_i &\propto s_i \left\{ \left[\frac{\alpha}{r(1+\tau_i^k)} \right]^{\alpha(1-\mu)\gamma} \left[\frac{\beta}{q(1+\tau_i^l)} \right]^{\beta(1-\mu)\gamma} \left[\frac{1-\alpha-\beta}{w(1+\tau_i^n)} \right]^{(-\alpha-\beta)(1-\mu)\gamma+\mu\gamma-1} \left[\frac{1}{z(1+\tau_i^x)} \right]^{\mu\gamma} \right\}^{\frac{1}{1-\gamma}}, \\
x_i &\propto s_i \left\{ \left[\frac{\alpha}{r(1+\tau_i^k)} \right]^{\alpha(1-\mu)\gamma} \left[\frac{\beta}{q(1+\tau_i^l)} \right]^{\beta(1-\mu)\gamma} \left[\frac{1-\alpha-\beta}{w(1+\tau_i^n)} \right]^{(1-\mu)\gamma(1-\alpha-\beta)} \left[\frac{1}{z(1+\tau_i^x)} \right]^{\mu\gamma+1-\gamma} \right\}^{\frac{1}{1-\gamma}}.
\end{aligned}$$

In equilibrium, the allocation of factor inputs depends not only on individual TFP level but also on all four distortions. Substituting k_i, l_i, n_i and x_i into a farm's technology (1) gives each farm's equilibrium output supply, y_i , which is proportional to

$$y_i \propto s_i \left\{ \left[\frac{\alpha}{r(1+\tau_i^k)} \right]^{\alpha(1-\mu)} \left[\frac{\beta}{q(1+\tau_i^l)} \right]^{\beta(1-\mu)} \left[\frac{1-\alpha-\beta}{w(1+\tau_i^n)} \right]^{(1-\alpha-\beta)(1-\mu)} \left[\frac{\mu}{z(1+\tau_i^x)} \right]^{\mu} \right\}^{\frac{\gamma}{1-\gamma}}. \quad (3)$$

Equilibrium output of a farm also depends on its TFP and the distortions it faces. The first order conditions give the marginal revenue product of capital (4), land (5), labor (6) and intermediate inputs (7), each of which is proportional to the specific revenue-factor ratio:

$$\text{MRPK}_i \equiv (1-\mu)\gamma\alpha \frac{py_i}{k_i} = (1+\tau_i^k)r \quad (4)$$

$$\text{MRPL}_i \equiv (1-\mu)\gamma\beta \frac{py_i}{l_i} = (1+\tau_i^l)q \quad (5)$$

$$\text{MRPN}_i \equiv (1-\mu)\gamma(1-\alpha-\beta) \frac{py_i}{n_i} = (1+\tau_i^n)w \quad (6)$$

$$\text{MRPX}_i \equiv \gamma\mu \frac{py_i}{x_i} = (1+\tau_i^x)z \quad (7)$$

The fact that distortions affect resource allocations shows up in the differences in the marginal revenue products. If $(1 + \tau_i^k) > 1$ (which implies a tax), then before tax marginal revenue product of capital must be lower than in farms that do not face taxes. Similarly, if $(1 + \tau_i^k) < 1$ (which implies a subsidy), then before tax marginal revenue product of capital must be higher than those farms that do not receive subsidies. Consider two farms, i and j : in the absence of distortions, the relative allocation of any factor, say, intermediate inputs, is given by

$$\frac{x_i}{x_j} \equiv \frac{s_i}{s_j}. \quad (8)$$

The more productive farm uses more intermediate inputs. With distortions, the relative allocation is given by

$$\frac{x_i}{x_j} \equiv \frac{s_i}{s_j} \left\{ \left[\frac{(1 + \tau_j^k)}{(1 + \tau_i^k)} \right]^{\alpha(1-\mu)\gamma} \left[\frac{(1 + \tau_j^l)}{(1 + \tau_i^l)} \right]^{\beta(1-\mu)\gamma} \left[\frac{(1 + \tau_j^n)}{(1 + \tau_i^n)} \right]^{(1-\mu)\gamma(1-\alpha-\beta)} \left[\frac{(1 + \tau_j^x)}{(1 + \tau_i^x)} \right]^{\mu\gamma+1-\gamma} \right\}^{\frac{1}{1-\gamma}}. \quad (9)$$

The relative allocation of intermediate inputs depends on individual TFP and all four distortions (taxes) as in equation (9). This is true for all four factors and has implications for equilibrium allocations—farms no longer receive resources proportional to individual TFPs. A farm even with high s but faced with distortions in the form of taxes uses proportionately less factor inputs than a farm with low s —henceforth, the misallocation. The factor ratios also vary across farms and are given by

$$\begin{aligned}
\text{capital land ratio} \quad \frac{k_i}{l_i} &= \frac{\alpha}{\beta} \frac{(1+\tau_i^l)}{(1+\tau_i^k)} \frac{q}{r}, \\
\text{capital-labor ratio} \quad \frac{k_i}{n_i} &= \frac{\alpha}{1-\alpha-\beta} \frac{(1+\tau_i^n)}{(1+\tau_i^k)} \frac{w}{r}, \\
\text{land-labor ratio} \quad \frac{l_i}{n_i} &= \frac{\beta}{1-\alpha-\beta} \frac{(1+\tau_i^n)}{(1+\tau_i^l)} \frac{w}{q}, \\
\text{capital-input ratio} \quad \frac{k_i}{x_i} &= \frac{\alpha(1-\mu)}{\mu} \frac{(1+\tau_i^x)}{(1+\tau_i^k)} \frac{z}{r}, \\
\text{labor-input ratio} \quad \frac{n_i}{x_i} &= (1-\alpha-\beta) \frac{(1-\mu)}{\mu} \frac{(1+\tau_i^x)}{(1+\tau_i^n)} \frac{z}{w},
\end{aligned}$$

and the resource constraints are: $K = \sum_{i=1}^M k_i$, $L = \sum_{i=1}^M l_i$, $N = \sum_{i=1}^M n_i$, $X = \sum_{i=1}^M x_i$ and $Y = \sum_{i=1}^M y_i$.

Next, to find an expression for the aggregate TFP (defined as $\text{TFP}^{\text{distorted}}$) as a function of distortions I do the following: Since aggregate demands of all four factors must add up to the four resource constraints, I divide each of the farm-level factor demands (k_i , l_i , n_i and x_i) by aggregate demand (K , L , N and X) respectively. Then, I divide and multiply K , L , N and X (now in terms of distortions that all farms face) by pY so that I can rearrange and express the farm-level factor demands as functions of the weighted average of the value of the marginal product of capital ($\overline{\text{MRPK}}$), of land ($\overline{\text{MRPL}}$), of labor ($\overline{\text{MRPN}}$) and of intermediate inputs ($\overline{\text{MRPX}}$) respectively.¹⁶ Therefore, each farm's allocations of its factors are given by

$$k_i = \left[\frac{\frac{py_i}{(1+\tau_i^k)} \frac{1}{pY}}{\sum \frac{py_i}{(1+\tau_i^k)} \frac{1}{pY}} \right] K = \overline{\text{MRPK}} \left[\frac{\frac{py_i}{(1+\tau_i^k)} \frac{1}{pY}}{r} \right] K \quad (10)$$

$$l_i = \left[\frac{\frac{py_i}{(1+\tau_i^l)} \frac{1}{pY}}{\sum \frac{py_i}{(1+\tau_i^l)} \frac{1}{pY}} \right] L = \overline{\text{MRPL}} \left[\frac{\frac{py_i}{(1+\tau_i^l)} \frac{1}{pY}}{q} \right] L \quad (11)$$

¹⁶ $\overline{\text{MRPK}} = \frac{r}{\sum \frac{py_i}{(1+\tau_i^k)} \frac{1}{pY}}$; $\overline{\text{MRPL}} = \frac{q}{\sum \frac{py_i}{(1+\tau_i^l)} \frac{1}{pY}}$; $\overline{\text{MRPN}} = \frac{w}{\sum \frac{py_i}{(1+\tau_i^n)} \frac{1}{pY}}$; $\overline{\text{MRPX}} = \frac{z}{\sum \frac{py_i}{(1+\tau_i^x)} \frac{1}{pY}}$.

$$n_i = \left[\frac{\frac{py_i}{(1+\tau_i^n)} \frac{1}{pY}}{\sum \frac{py_i}{(1+\tau_i^n)} \frac{1}{pY}} \right] N = \overline{\text{MRPN}} \left[\frac{\frac{py_i}{(1+\tau_i^n)} \frac{1}{pY}}{w} \right] N \quad (12)$$

$$x_i = \left[\frac{\frac{py_i}{(1+\tau_i^x)} \frac{1}{pY}}{\sum \frac{py_i}{(1+\tau_i^x)} \frac{1}{pY}} \right] X = \overline{\text{MRPX}} \left[\frac{\frac{py_i}{(1+\tau_i^x)} \frac{1}{pY}}{z} \right] X \quad (13)$$

where K, L, N and X are aggregate capital, land, labour and intermediate inputs respectively. Substituting k_i, l_i, n_i and x_i into equation (1) gives y_i as a function of its TFP (s_i) and weighted marginal products.

$$y_i = \left\{ s_i \left(\frac{1}{Y} \right)^{\frac{\gamma}{1-\gamma}} \left[(\overline{\text{MRPK}})^\alpha (\overline{\text{MRPL}})^\beta (\overline{\text{MRPN}})^{1-\alpha-\beta} \right]^{\frac{(1-\mu)\gamma}{1-\gamma}} (\overline{\text{MRPX}})^{\frac{\mu\gamma}{1-\gamma}} \left(K^\alpha L^\beta N^{1-\alpha-\beta} \right)^{\frac{(1-\mu)\gamma}{1-\gamma}} X^{\frac{\mu\gamma}{1-\gamma}} \left(\left[\frac{1}{(1+\tau_i^k)r} \right]^\alpha \left[\frac{1}{(1+\tau_i^l)q} \right]^\beta \left[\frac{1}{(1+\tau_i^n)w} \right]^{1-\alpha-\beta} \right)^{(1-\mu)\frac{\gamma}{1-\gamma}} \left[\frac{1}{(1+\tau_i^x)z} \right]^\mu \right\}^{\frac{\gamma}{1-\gamma}}$$

Since $Y = \sum y_i$

$$Y = \left\{ \frac{\sum s_i \left(\left[\frac{\overline{\text{MRPK}}}{(1+\tau_i^k)r} \right]^\alpha \left[\frac{\overline{\text{MRPL}}}{(1+\tau_i^l)q} \right]^\beta \left[\frac{\overline{\text{MRPN}}}{(1+\tau_i^n)w} \right]^{1-\alpha-\beta} \right)^{\frac{(1-\mu)\gamma}{1-\gamma}} \left[\frac{\overline{\text{MRPX}}}{(1+\tau_i^x)z} \right]^{\frac{\mu\gamma}{1-\gamma}}}{M} \right\}^{1-\gamma} \cdot \underbrace{\left(K^\alpha L^\beta N^{1-\alpha-\beta} \right)^{(1-\mu)\gamma} X^{\mu\gamma} M^{1-\gamma}}_{\text{Aggregate Resources}}$$

$$Y = \text{TFP} \left[\left(K^\alpha L^\beta N^{1-\alpha-\beta} \right)^{(1-\mu)} X^\mu \right]^\gamma M^{1-\gamma}. \quad (14)$$

Aggregate output (Y) is a function of K, L, N, X, M (the number of farms) and aggregate TFP (which is itself a function of the distribution of individual TFPs and distortions). Aggregate TFP in equation (14) is inefficient or distorted because of the farm-specific distortions. To find an exact

expression for it from individual farm-level TFPs I adopt two measures of productivity, commonly used in literature (Foster et al, 2008). The first measure that I use is *physical productivity* (TFPQ), which is revenue deflated by a farm specific deflator, and is defined as

$$\text{TFPQ}_i = s_i^{1-\gamma} = \frac{y_i}{\left[\left(k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta} \right)^{1-\mu} x_i^\mu \right]^\gamma}. \quad (15)$$

TFPQ_i estimates difference in physical productivity across farms; however, if an aggregate price deflator is used, then farm-level TFP will confound higher prices with higher productivity. The second measure that I use is *revenue productivity* (TFPR), which is defined as

$$\text{TFPR}_i = \frac{py_i}{\left(k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta} \right)^{1-\mu} x_i^\mu} = \left(\frac{py_i}{k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta}} \right)^{1-\mu} \left(\frac{py_i}{x_i} \right)^\mu. \quad (16)$$

TFPR is useful because farm-specific distortions can be measured by it and variation of TFPR_i is a measure of misallocation. It can be estimated by substituting revenue-capital (10), revenue-land (11), revenue-labor (12) and revenue-intermediate inputs (13) ratios, that are functions of farm-level distortions, into (16) giving

$$\text{TFPR}_i = \left\{ \left[\frac{r(1+\tau_i^k)}{\gamma(1-\mu)\alpha} \right]^\alpha \left[\frac{q(1+\tau_i^l)}{\gamma(1-\mu)\beta} \right]^\beta \left[\frac{w(1+\tau_i^n)}{\gamma(1-\mu)(1-\alpha-\beta)} \right]^{1-\alpha-\beta} \right\}^{1-\mu} \left(\frac{z(1+\tau_i^x)}{\gamma\mu} \right)^\mu \quad (17)$$

$$= \left\{ \left[\frac{\text{MRPK}_i}{(1-\mu)\gamma\alpha} \right]^\alpha \left[\frac{\text{MRPL}_i}{(1-\mu)\gamma\beta} \right]^\beta \left[\frac{\text{MRPN}_i}{(1-\mu)\gamma(1-\alpha-\beta)} \right]^{1-\alpha-\beta} \right\}^{1-\mu} \left(\frac{\text{MRPX}_i}{\gamma\mu} \right)^\mu, \quad (18)$$

which is proportional to a geometric average of a farm's marginal revenue products of capital, land, labor and intermediate inputs. TFPR_i is a summary measure of farm-specific distortions. Under an efficient allocation (no distortions), a farm with high TFPQ should use more resources, produce

more until its TFPR matches that of a small farm. However, with distortions TFPR varies across farms. For instance, if a farm's TFPR is large, then the constraints it faces alter its effective marginal products, making it smaller than optimally sized, simply because it uses less resources.

Combining the expressions for TFPR_i and TFPQ_i the measure of distorted aggregate TFP is

$$\text{TFP}^{\text{distorted}} = \left\{ \sum_{i=1}^M \frac{\left[\text{TFPQ}_i \left(\frac{\overline{\text{TFPR}}}{\text{TFPR}_i} \right)^\gamma \right]^{\frac{1}{1-\gamma}}}{M} \right\}^{1-\gamma}, \quad (19)$$

where $\overline{\text{TFPR}}$ is a geometric average of the average marginal revel product of capital, land labor and intermediate inputs.¹⁷ If there are no farm specific distortions, then equalization of marginal products means that $\overline{\text{TFPR}} = \text{TFPR}_i$ and the aggregate measure of TFP (equation 19) becomes

$$\text{TFP}^{\text{efficient}} = \left\{ \frac{\sum_{i=1}^M \text{TFPQ}_i^{\frac{1}{1-\gamma}}}{M} \right\}^{1-\gamma}, \quad (20)$$

which is the weighted average of all farms' individual TFPQs. Finally, the ratio of the distorted and efficient TFP gives a measure of TFP gain given by

$$\text{TFP}^{\text{gain}} = \left\{ \frac{\left\{ \sum_{i=1}^M \text{TFPQ}_i^{\frac{1}{1-\gamma}} \right\}^{1-\gamma}}{\left\{ \sum_{i=1}^M \left[\text{TFPQ}_i \left(\frac{\overline{\text{TFPR}}}{\text{TFPR}_i} \right)^\gamma \right]^{\frac{1}{1-\gamma}} \right\}^{1-\gamma}} - 1 \right\} 100\%. \quad (21)$$

¹⁷ $\overline{\text{TFPR}} = \left[\left\{ \left[\frac{\overline{\text{MRPK}}}{\gamma(1-\mu)\alpha} \right]^\alpha \left[\frac{\overline{\text{MRPL}}}{\gamma(1-\mu)\beta} \right]^\beta \left[\frac{\overline{\text{MRPN}}}{\gamma(1-\mu)(1-\alpha-\beta)} \right]^{1-\alpha-\beta} \right\} \right]^{1-\mu} \left(\frac{\overline{\text{MRPX}}}{\gamma\mu} \right)^\mu$

2.4 Data Set for Bangladesh Agriculture and Distortions

The data for Bangladesh agriculture are from Bangladesh Household Income and Expenditure Survey (HIES), conducted by the government monitored Bangladesh Bureau of Statistics (BBS). The HIES is a survey done on a regular basis that provides important data like income, expenditure, consumption and poverty situation.¹⁸ Data are collected through surveys at field-level and processed at BBS headquarters. The first survey was done in 1973-74 and the last, which I use, in 2010. The 2010 survey covers 12240 households (based on 612 primary sampling units from 16 strata) representing a population of more than 150 million living in 7 regions (known as Divisions).¹⁹ The households that are employed in manufacturing and agriculture are entrepreneurs, self-employed and/or workers; 7840 households live in rural and 4400 in urban areas; 6768 work in agriculture and the rest in non-agriculture. So, both manufacturing and agricultural activities take place in rural and urban areas. Agricultural activities take place in 5352 farms.

To calculate the defined farm-level gross TFPQ I need capital, land, labor, intermediate inputs and a set of weighted prices. Each farm produces different types of crops using capital, land, labor and intermediate inputs. Farms that do not report any usage of land and capital are dropped. Aggregate capital is $K = \sum_{i=1}^M k_i$.²⁰ Only 1124 out of 5352 farms are reported to own assets in some form of machinery, though it is unlikely that farms do not own any assets.²¹ Operators owning traditional farms that use hand tools probably did not report assets. It may also be the case that many operators share or rent equipment (such as tractor, power tiller, tube well, etc.) which is not explicitly mentioned in the survey. Due to missing information the total value of capital at farm

¹⁸ Also disability, micro-credit, migration and remittance, crisis and crisis management.

¹⁹ Same households are not surveyed across time.

²⁰ Capital and asset are interchangeably used.

²¹ Assets are: tractor, thresher, power tiller, power pump, hand pump, plough and yoke, deep tube well, shallow tube well, sprayer, husking machine, ginning machine.

level is not measured at weighted prices but is the simple addition of all reported assets. Aggregate land is $L = \sum_{i=1}^M \sum_{j=1}^J l_{ij}$, where J = number of crops a farm produces, M = number of farms; i is an individual farm and j a unique crop.²² After trimming the 0.5% tails of the $\log(\text{TFPR}_i/\overline{\text{TFPR}})$ distribution to make the results robust to outliers, my sample consists of 1112 farms that report the usage of all four factor inputs.

Measuring effective labor hours of a farm is not straightforward as farms only report annual expenditure on workers. The farm operator and family members probably work on the same farm without any direct monetary compensation. So, one crucial assumption that I make is that, if the members of a household identify as agricultural workers, I count them as family workers employed on the same farm. Total effective labor hours is the summation of the hours imputed from the reported annual expenditure on hired workers $w \cdot n_0$ (where w = hourly wage, n_0 = total effective annual hours, supplied by hired workers) and the hours of family members (n_1) working on the same farm. Generally, there are three types of workers in agriculture: self-employed, workers who are paid daily in cash and/or in kind, and workers who are paid monthly.²³ To impute hourly wage (w) I use information from the household survey. In the survey, households, if daily waged in agriculture, are asked about the average daily cash and non-cash wages that they receive, the average number of months in a year, the average number of days in a month and the average number of hours per day they work. Households, if monthly waged, are asked about the monthly remuneration and the yearly cash benefits that they receive. So, to find n_0 I do the following: First, for those who are daily waged, I divide their average daily cash and in-kind wage by the average number of hours per day they work. Second, for those who are monthly waged, I divide their yearly benefits by the

²²There are 39 categories of crops and non-crops (such as vegetables, crop by-products and fruits); therefore, $k = 39$.

²³Workers who receive monthly remuneration may also receive yearly benefits. I divide yearly benefits by the number of months they work in a year and add to their regular monthly remuneration.

number of months they work, add to it the monthly wage and then divide all of it by the average number of days a month and the average number of hours per month they work. Finally, I take a simple regional average of the wages of agricultural workers to find hourly wage (w). So, effective labor hour (n_i) of a farm is $(w \cdot n_{0,i}) / w + (h - 1) \cdot n_{1,i}$ where h is total members of a household including the operator. Aggregate labor hours is $N = \sum_{i=1}^M n_i$.

Next, I find the value of intermediate inputs. According to the survey, six intermediate inputs are relevant for agriculture: *crop seedling*, *chemical fertilizer*, *compost fertilizer*, *irrigation expenses*, *insecticides* and *electricity (and fuel)*. For the first 3 intermediate inputs, each input (x_{id}) is multiplied by its weighted price (z_d^w).²⁴ For the last 3 intermediate inputs, a farm reports only annual expenses (Z_{id}) that it incurs; therefore, the value of each farm's intermediate inputs (x_i) is $(\sum_{d=1}^3 z_d^w x_{id} + \sum_{d=4}^6 Z_{id})$. Aggregate intermediate inputs is $X = \sum_{i=1}^M x_i$. A farm produces different crops ($j = 1 \dots J$) using capital, land, labor and intermediate inputs. The survey has information about crop variety, their quantities and unit prices. Each variety (q_{ij}) that a farm produces is multiplied by its weighted price p_j^w ; therefore, the gross value of each farm's output (y_i) is the summation of the products it produces $(\sum_{j=1}^J p_j^w q_{ij})$.²⁵ Aggregate gross output is $Y = \sum y_i$. Finally, all variables (k_i, l_i, n_i, x_i, y_i) are divided by the number of hours of a farm operator, which, since imputed from section 4 of the survey, varies according to division.

The distortions can be isolated from the marginal revenue products. From the marginal revenue product of capital (4), the capital distortion can be inferred as

²⁴ $z_d^w = \sum_{i=1}^m \left[z_{id} \left(\frac{x_{id}}{\sum_{i'=1}^m x_{i'd}} \right) \right]$, where z_d^w is the weighted price of a specific intermediate input x_{id} . z_{id} is the unit price a farm incurs for the input d ; $x_{id} / \sum_{i'=1}^m x_{i'd}$ is its share of input in total quantity used by all farms.

²⁵ $p_j^w = \sum_{i=1}^M \left[p_{ij} \left(\frac{q_{ij}}{\sum_{i'=1}^M q_{i'j}} \right) \right]$, where p_j^w is the weighted price of a specific crop j . p_{ij} is the unit price a farm gets for that crop, $q_{ij} / \sum_{i'=1}^M q_{i'j}$ is its share of production in total quantity produced. 39 types of crops are listed; therefore, $J = 39$.

$$(1 + \tau_i^k) = \frac{(1 - \mu)\gamma\alpha}{r} \frac{py_i}{k_i}. \quad (22)$$

From the marginal revenue product of land (5), the land distortion can be inferred as

$$(1 + \tau_i^l) = \frac{(1 - \mu)\gamma\beta}{q} \frac{py_i}{l_i}. \quad (23)$$

From the marginal revenue product of labor (6), the labor distortion can be inferred as

$$(1 + \tau_i^n) = \frac{(1 - \mu)\gamma(1 - \alpha - \beta)}{w} \frac{py_i}{n_i}. \quad (24)$$

From the marginal revenue product of intermediate inputs (7), the intermediate inputs distortion can be inferred as

$$(1 + \tau_i^x) = \frac{\gamma\mu}{z} \frac{py_i}{x_i}. \quad (25)$$

It does not matter what values for r, q, w and z are chosen because, in the calculation of distorted TFP (19), the ratio of $\overline{\text{TFPR}}$ and TFPR_i cancels out the factor prices.²⁶ Common to all distortions is the span-of-control parameter (γ); the higher the span the greater is the effect of distortions. The measures of distortions, efficient and distorted TFP are all very sensitive to the parameter values, which determine the elasticities. I do not set the elasticities on the basis of factor income shares in Bangladesh's agriculture because they would confound elasticities and distortions. As benchmark, I use U.S. shares reported in Valentinyi and Herrendorf (2008) because the agriculture of the United States is relatively undistorted. They show that the net capital income share is 0.36, land 0.18 and

²⁶For convenience, $r = q = w = z = 1$.

labor 0.24. Therefore, for the benchmark model, I choose $\gamma = 0.78, \alpha = 0.4615, \beta = 0.2308$.²⁷ These parameter values are consistent with those used in standard literature, such as, Restuccia and Santaaulàlia (2015). Intermediate inputs income share in gross output is 0.40, so $\mu = 0.5128$. With these parameter values, in gross output, the share of capital is 17.54%, of land 8.77%, of labor 11.69% and of intermediate goods 40%.

2.5 Evidence of Misallocation and Gains from Reallocation

For any technology, the efficient allocation of a factor is such that the relatively more productive farmers would operate with more capital, land, labor and intermediate inputs and produce more output. In other words, at an aggregate level the correlation between any factor and productivity should be positive and strong. On the contrary, if the correlation is not strong, then farms are not optimally using that factor suggesting that it is being misallocated. Using the measure of productivity given by equation 15 and the parameter values mentioned in the previous section, a simple check of the actual allocation of each factor by farm productivity across farms can reveal the extent of misallocation in agriculture, shown in Figure 2.3.

²⁷ $\gamma = 0.78$ reflects wide span-of-control.

Figure 2.3 (Capital and Land by Productivity)

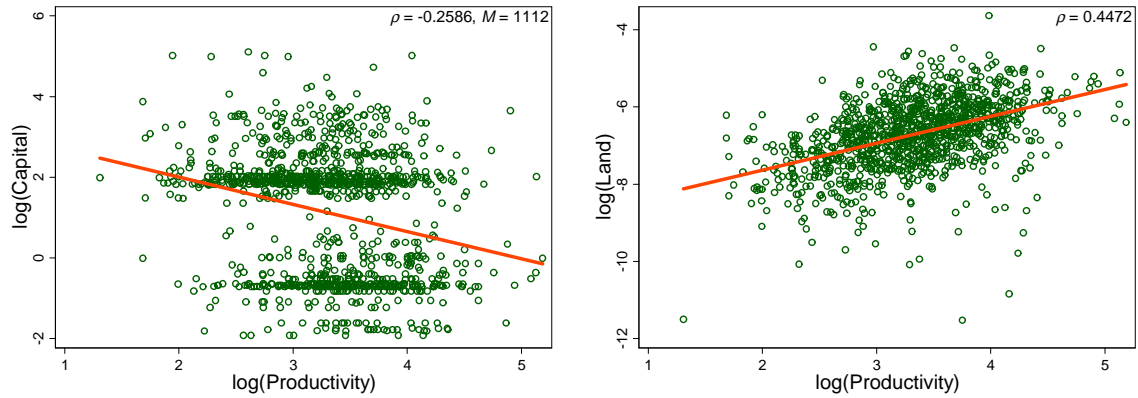
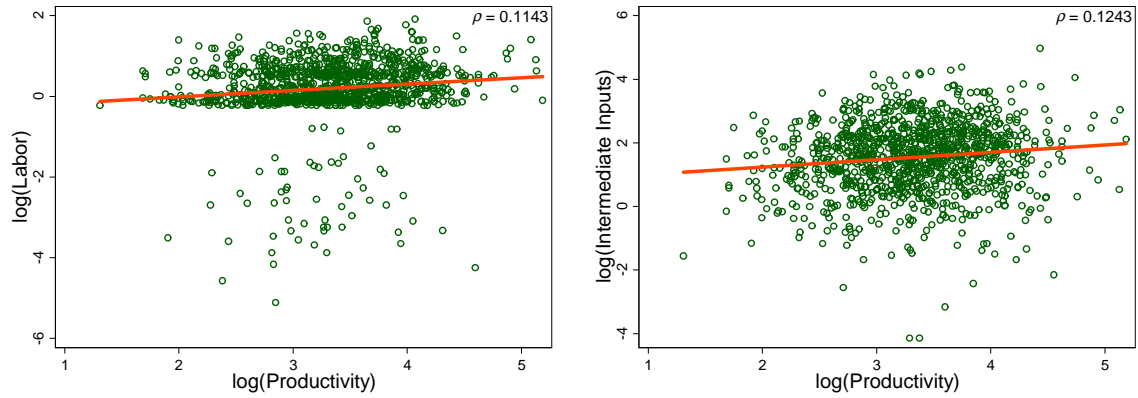


Figure 2.3 Continued (Labor and Intermediate Inputs by Productivity)



In the top left panel of Figure 2.3 the negative correlation of 0.2586 between capital (per farm operator hour) and TFPQ shows that the more productive farms do not use more capital. In the top right panel, the positive correlation of 0.4472 between land and productivity shows that compared to other factors, the more productive farms use more land. In the bottom panel of Figure 2.3 the correlations of 0.1243 between intermediate inputs and TFPQ and 0.1143 between labor and

TFPQ are too small to conclude that the more productive farms use more of these two factors; the actual allocations of labor and intermediate inputs in farms are not significantly related to farm productivity. There are some farms where the hours of hired workers (and family members) are less than that of a farm operator (owner).²⁸ For capital there are two groups of farms (two clusters), each with low correlation still indicating misallocation. These simple plots show that capital is most misallocated and land the least.

In standard models like Lucas (1978) and Hopenhayn (1992), in the absence of distortions, marginal products of factors should be equalized across farms which implies equalization of average products. In other words, if there is no misallocation at an aggregate level, then average products (proportional to marginal products) should not vary by productivity. Higher productivity units should produce more and command more factors. To check whether this holds, I plot each farm's average product against its productivity in Figure 2.4.²⁹

²⁸Farm operator hours per year: 1885 in Borishal; 1897 in Chittagong; 1970 in Dhaka; 1675 in Khulna; 2020 in Rajshahi; 2035 in Rongpur; 2256 in Sylhet. Farm worker hours: 1604 in Borishal; 1510 in Chittagong; 1565 in Dhaka; 1515 in Khulna; 1790 in Rajshahi, 1771 in Rongpur; 1797 in Sylhet.

²⁹For instance, given Cobb-Douglas production function, average product of capital is proportional to marginal product of capital. The rest follows similarly.

Figure 2.4: Average Product by Productivity

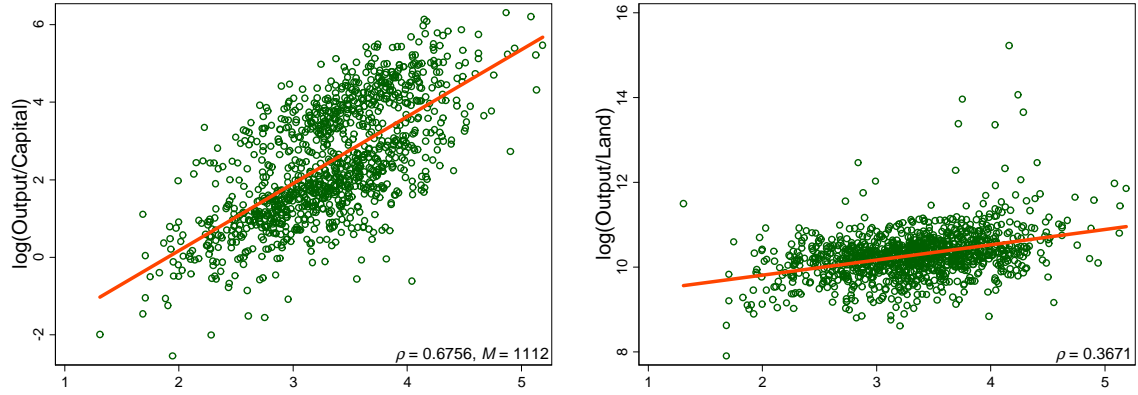
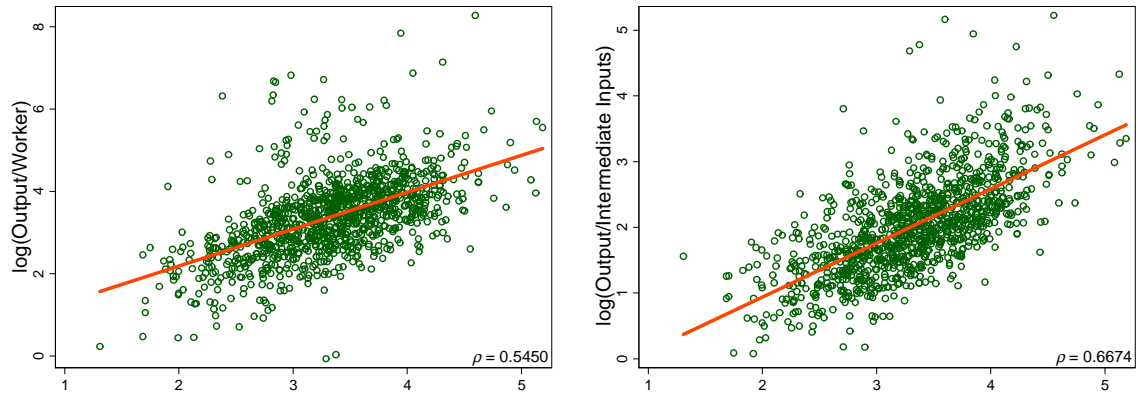


Figure 2.4: (Continued)



In all four panels the positive correlation between average product and TFP is positive and significantly different from zero; in other words, average product of these factors are not equated across farms indicating misallocation. The correlations between average product of capital, land, labor, intermediate inputs and farm level TFP are 0.6756, 0.3671, 0.5450 and 0.6674 respectively. These plots convey the same finding as Figure 2.3 that factors are misallocated; however, it appears

that capital and intermediate inputs are most misallocated and land the least.

In summary, the actual allocation of capital and intermediate inputs across farms in Bangladesh are unrelated to farm productivity. This is consistent with what is observed in the survey (by Barkat et al., 2010) as farmers face financial constraints and have to routinely deal with fertilizer crises. My interpretation of this finding is that capital market distortions not only affect capital allocations directly but they also indirectly affect the allocations of intermediate inputs. Figure 2.3 and 2.4 provide strong evidence of capital misallocation across farms in agriculture; evidence of land misallocation is relatively very weak; intermediate inputs and labor appear just as misallocated, but the productivity effects are larger for intermediate inputs.

In the left panel of Figure 2.5, TFPQ is marginally less dispersed than TFPR (both relative to sectoral means weighted by their gross value shares).³⁰ TFPQ and TFPR are highly correlated with a coefficient of 0.9498, shown in the right panel; in other words, the more productive farms are subject to higher distortions adversely affecting aggregate TFP. The covariance between TFPR and TFPQ is 0.3468. If the more productive farms are affected by greater misallocations (implying a stronger positive correlation between TFPR and TFPQ), then TFP gains from equalizing TFPR should be large which I investigate next.

³⁰Sectoral weighted $\overline{\text{TFPQ}} = \sum \theta_i \cdot \text{TFPQ}_i$ where $\theta_i = \frac{py_i}{\sum py_i}$.

Figure 2.5: Distribution of Agricultural TFPQ and TFPR

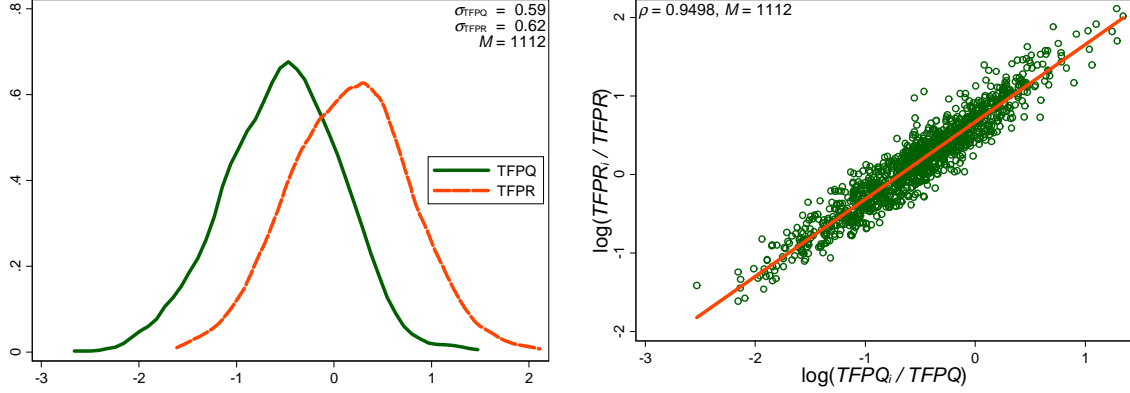


Table 2.2: Dispersion of $\log(\text{TFPQ})$ across Farms and Manufacturing Plants

Statistic	Farms			Manufacturing Plants		
	Bangladesh 2010	Malawi 2010	USA 1990	USA 1997	China 2005	India 1994
SD	0.59	0.95	0.80	0.84	0.95	1.23
75-25	0.83	1.15	1.97	1.17	1.28	1.60
90-10	1.52	2.38	2.50	2.18	2.44	3.11
M	1112	8009	AR(2014)	194669	211304	41006

The first column reports statistics for the untrimmed household-farm productivity distribution (deviations of $\log(\text{TFPQ})$ from aggregate mean weighted by gross-value-added shares) from the micro data in Bangladesh. The 3rd column reports statistics from farms in Restuccia and Santaella-Llopis (2015). The 4th column reports US farm productivity statistics from the calibrated distribution in Adamopoulos and Restuccia (2014a) to US farm-size data. The 5th, 6th and 7th columns report manufacturing plants statistics in Hsieh and Klenow (2009). SD is the standard deviation of $\log(\text{TFPQ})$ from its aggregate mean, 75-25 is the difference between the 75th and 25th percentiles, and 90-10 the 90 to 10 percentile difference in productivity. M is the number of observations.

The physical productivity dispersion across Bangladeshi farms is less than the physical productivity dispersion of agricultural farms reported in Malawi and manufacturing plants in the USA, China and India, and it is consistent with several measures of dispersion of $\log(\text{TFPQ})$. This implies that in the actual allocation of resources there are severe misallocations from distortions—the more productive farms systematically face more distortions. The efficient allocation requires that

marginal products are equated across farms, implying $\overline{\text{TFPR}} = \text{TFPR}_i$, in which case the efficient allocations of the four factors in terms of productivity are:

$$k_i^{\text{efficient}} = \frac{\text{TFPQ}_i^{\frac{1}{1-\gamma}}}{\sum_{i=1}^M \text{TFPQ}_i^{\frac{1}{1-\gamma}}} K, \quad l_i^{\text{efficient}} = \frac{\text{TFPQ}_i^{\frac{1}{1-\gamma}}}{\sum_{i=1}^M \text{TFPQ}_i^{\frac{1}{1-\gamma}}} L,$$

$$n_i^{\text{efficient}} = \frac{\text{TFPQ}_i^{\frac{1}{1-\gamma}}}{\sum_{i=1}^M \text{TFPQ}_i^{\frac{1}{1-\gamma}}} N, \quad x_i^{\text{efficient}} = \frac{\text{TFPQ}_i^{\frac{1}{1-\gamma}}}{\sum_{i=1}^M \text{TFPQ}_i^{\frac{1}{1-\gamma}}} X.$$

The extent of misallocations is illustrated in Figure 2.6 where I show the difference between actual and efficient allocation of capital, land, labor and intermediate goods. If resources were allocated efficiently, there would be strong positive correlations (of 1) between each factor and farm productivity, implying constant marginal productivity across farms.

Figure 2.6: Efficient versus Real Allocation by Productivity

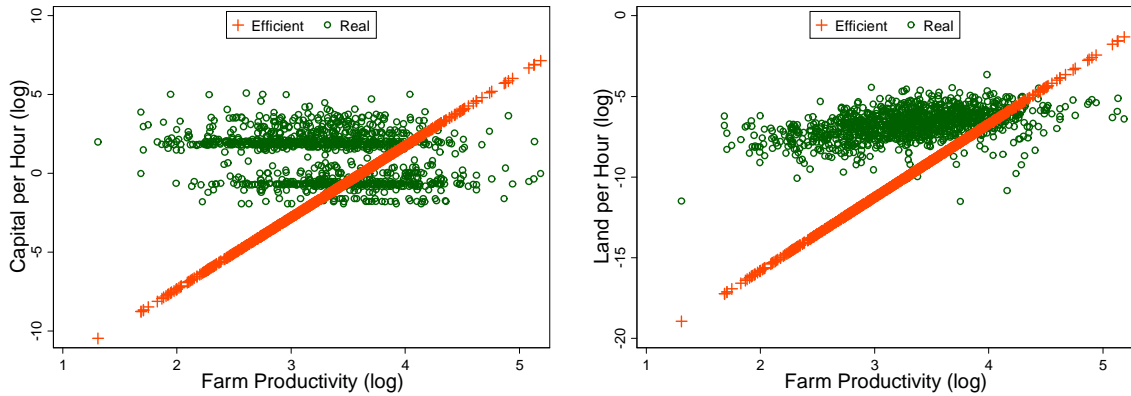
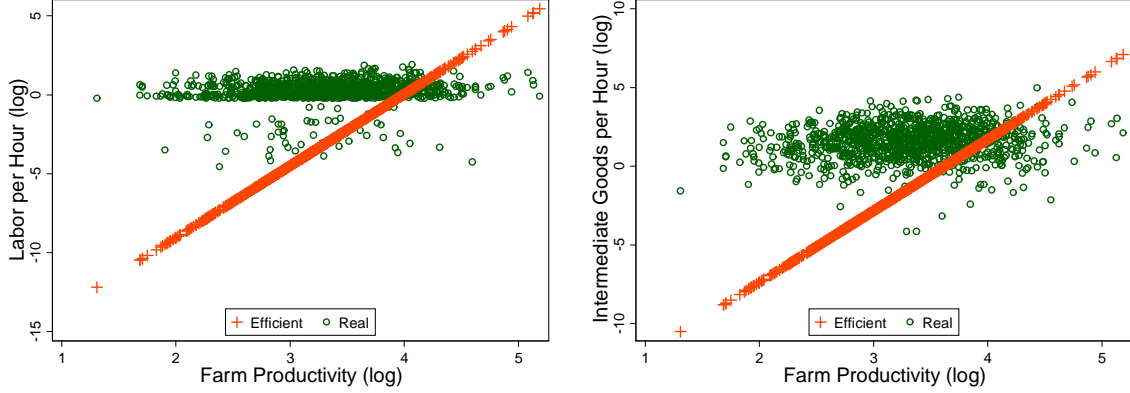


Figure 2.6 (Continued)



The output loss is the ratio of actual agricultural output $\sum py_i$ to efficient aggregate agricultural output $Y = \text{TFP}^{\text{efficient}} \left[(K^\alpha L^\beta N^{1-\alpha-\beta})^{(1-\mu)} X^\mu \right]^\gamma M^{1-\gamma}$, where $\text{TFP}^{\text{efficient}}$ is obtained by equalizing TFPR across farms.³¹ The estimated output loss is 0.45, that is actual agricultural output is a mere 45% of the efficient output at the aggregate level. I use equation (21) to find the percent TFP gains after removing distortions and equalizing TFPR. The aggregate TFP gain is reported in Table 2.3.

Table 2.3: TFP Gain with All Types of Distortions				
$\gamma = 0.78, \alpha = 0.4615, \beta = 0.2308, \mu = 0.5128$				
M	Distortions	Efficient TFP	Distorted TFP	TFP ^{gain}
1112	$\tau^k, \tau^l, \tau^n, \tau^x \neq 0$	59.06	26.79	120.42%

If capital, land, labor and intermediate inputs were reallocated efficiently to maximize output, then Bangladesh's agricultural productivity would increase by nearly 120%. This gain is large and important for Bangladesh, but considerably smaller than 3.6-fold that Restuccia and Santaaulàlia

³¹ $\text{TFPR}_i = \overline{\text{TFPR}}$.

(2015) find for Malawi.³²

Next, I ask which type of distortions (capital, land, labor or intermediate inputs) contribute the most to TFP losses. Given that the average level of distortions is less important than the variation in idiosyncratic distortions (Bond et al, 2013), the greater dispersions in capital and intermediate inputs distortions than in land and labor distortions (Figure 2.7) suggest that capital and intermediate inputs distortions contribute to TFP losses by bigger magnitude. The correlations between TFPR and TFPQ also indicate the same: the correlation between TFPR with capital distortion and TFPQ is 0.6756 and between TFPR with intermediate inputs distortion and TFPQ is 0.6674. These correlations are larger than the correlation of 0.3671 between TFPR and TFPQ with land distortions, and of 0.5450 between TFPR and TFPQ with labor distortions. I do not isolate the partial effect of each type of distortion in the presence of all four types of distortions; instead, I assume farms are subject to only one type of distortion and then re-estimate TFP gains. For instance, if capital distortions were removed in the presence of capital distortions only (with all other distortions equal to zero), then, at an aggregate level, TFP gain would be larger than what it would be if land distortions were removed in the presence of land distortions only (with all other distortions equal to zero).³³ I do the same for all four distortions; the results are reported in Table 2.4 showing the percentage gain between the distorted and efficient TFP.³⁴ Capital and intermediate inputs distortions contribute the most to TFP losses in agriculture, whereas, losses from labor and land distortions are significantly much smaller.

³²Hsieh and Klenow (2009) find TFP gains of 100-160% in China's and India's manufacturing.

³³ $\tau^k \neq 0, \tau^l, \tau^n, \tau^x = 0$.

³⁴Another approach can be used to find TFP gains from removing one particular type of distortions. For instance, the TFP gains from removing capital distortions can be measured by $\frac{TFP_{\tau^l, \tau^n, \tau^x \neq 0}^{\text{distorted}}}{TFP_{\tau^k, \tau^l, \tau^n, \tau^x \neq 0}^{\text{distorted}}}$. Using this alternative approach the TFG gains from removing capital, land, labor and intermediate inputs are 34.64%, 2.20%, 6.53% and 35.59% respectively. Therefore, I confirm that the relative importance of the different factors does not change.

Figure 2.7: Distribution of TFPR of Each Factor

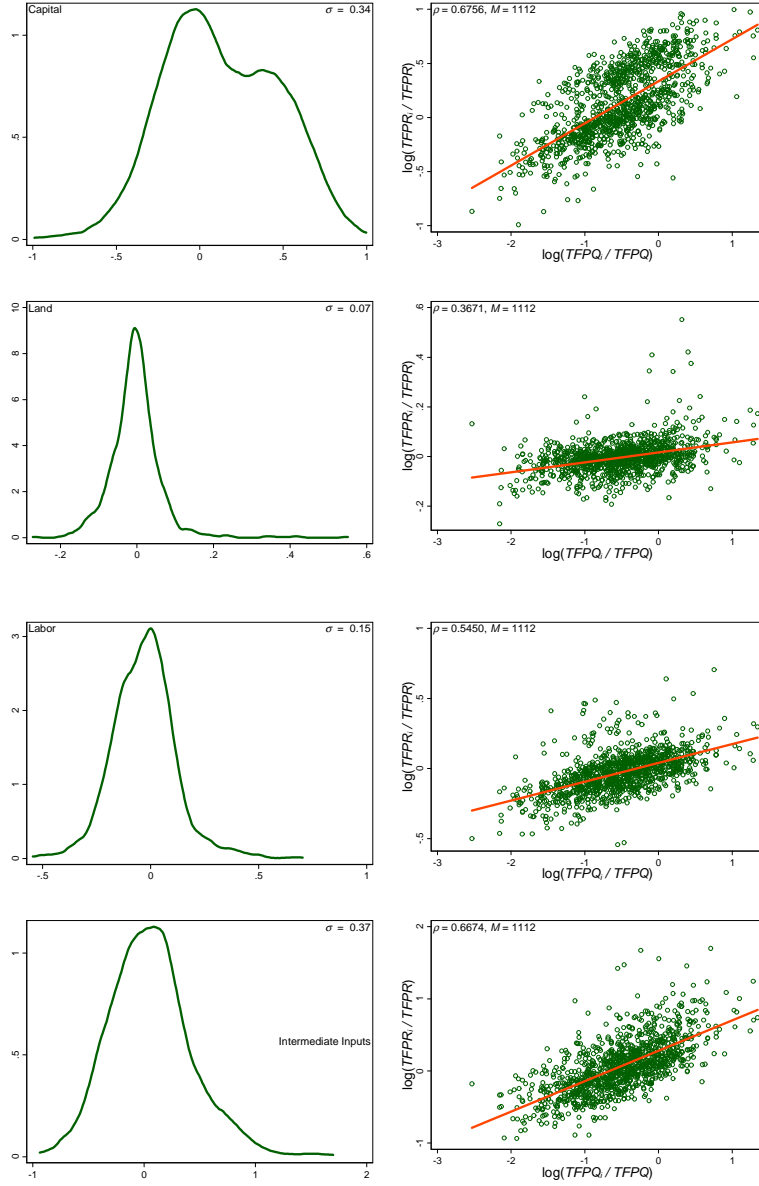


Table 2.4: TFP Gains with Each Type of Distortions

$\gamma = 0.78, \alpha = 0.4615, \beta = 0.2308, \mu = 0.5128$

M	Distortions	Efficient TFP	Distorted TFP	TFP ^{gain}
1112	$\tau^k \neq 0, \tau^l, \tau^n, \tau^x = 0$	59.06	40.36	46.32%
1112	$\tau^l \neq 0; \tau^k, \tau^n, \tau^x = 0$	59.06	55.87	5.70%
1112	$\tau^n \neq 0; \tau^k, \tau^l, \tau^x = 0$	59.06	53.63	10.11%
1112	$\tau^x \neq 0; \tau^k, \tau^l, \tau^n = 0$	59.06	45.87	45.87%

In order to find out the effect of distortions on the size of the farm, I divide farms into 5 classes: There are 60 farms in the class of "Landless" with land up to 0.49 acre; 250 in the class of "Marginal" with land above 0.49 and below 1.49; 253 in the class of "Small" with land above 1.49 and below 2.49; 454 in the class of "Medium" with land above 2.49 and below 7.49; 95 in the class of "Large" with land above 7.49. Within each class, I equate $\overline{\text{TFPR}} = \text{TFPR}_i$ and then estimate the TFP gains. Although TFP gain is the largest in the Marginal class and smallest in the Large class, shown in Table 2.5, it's evident that there is misallocation in all farm classes.

Table 2.5: TFP Gains of Different Farm Classifications					
Farm Size	Farm Class	Distortions	Efficient TFP	Distorted TFP	TFP ^{gain}
≤ 0.49	Landless	$\tau^k, \tau^l, \tau^n, \tau^x \neq 0$	44.92	20.10	123.50%
(0.49, 1.49]	Marginal	$\tau^k, \tau^l, \tau^n, \tau^x \neq 0$	38.12	16.10	136.75%
(1.49, 2.49]	Small	$\tau^k, \tau^l, \tau^n, \tau^x \neq 0$	40.82	20.45	99.59%
(2.49, 7.49]	Medium	$\tau^k, \tau^l, \tau^n, \tau^x \neq 0$	64.54	29.84	116.28%
> 7.49	Large	$\tau^k, \tau^l, \tau^n, \tau^x \neq 0$	77.84	40.24	93.44%

2.6 Further Robustness Results

In order to determine the effects of financial institutions on capital distortion I estimate a model that allows for variations in capital distortions among three groups of financial institutions: formal, micro-credit and informal. I select informal institutions as the base group; the dummy variables for the remaining groups are *Formal* and *MicroCredit*. The log of loan (normalized by labor input) and log of TFPQ (relative to weighted average) are added as controls; the model estimated is reported in Table 2.6.

Table 2.6: Effects of Financial Institutions

$$\log(\widehat{capital\ distortion}) = \begin{matrix} -4.484 \\ (-9.34) \end{matrix} - \begin{matrix} 0.227 \log(Loan) \\ (-2.55) \end{matrix} + \begin{matrix} 1.844 \log(TFPQ) \\ (14.67) \end{matrix}$$

$$- \begin{matrix} 0.706 Formal \\ (-3.07) \end{matrix} - \begin{matrix} 0.361 MicroCredit \\ (-1.60) \end{matrix}$$

$$R^2 = 0.49; M = 216$$

All of the coefficients, with the exception of *MicroCredit*, have significant t statistics. The t statistic for *MicroCredit* is -1.60, which is approximately significant at the 10% level. Holding TFPQ and loan fixed, farms that borrow from formal institutions are estimated to face 50.64% less capital distortions than those that borrow from informal institutions. Similarly, farms that borrow from micro credit institutions are estimated to face 30.30% less capital distortions than those that borrow from informal institutions.³⁵ Relative to formal institutions, farms that borrow from micro credit institutions face 34.50% less capital distortions. I also regress $\log(TFPR)$ on $\log(TFPQ)$ and find that the coefficients are positive and significant, as reported in Table 2.7.

Table 2.7: Elasticity of Different Farm Classifications				
Farm Size	Farm Class	M	Elasticity	Standard Error
	All	1112	0.9850	0.0097
≤ 0.49	Landless	60	0.9870	0.0347
(0.49, 1.49]	Marginal	250	1.1140	0.0153
(1.49, 2.49]	Small	253	1.1190	0.0154
(2.49, 7.49]	Medium	454	1.1185	0.0128
> 7.49	Large	95	1.0782	0.0304

Next, I consider a model of net output with capital, land and labor distortions (Table 2.8). The share of capital in net output is 0.36, of land 0.18 and of labor 0.24. The aggregate TFP gain is 202%. If distortions were removed in the presence of capital distortions only, then the TFP gain

³⁵The exact percentage is estimated as $100 * [\exp(\hat{\beta}) - 1]$.

would be 119%, again showing that capital is most misallocated.

Table 2.8: TFP Gains with 3 Factors				
$\gamma = 0.78, \alpha = 0.4615, \beta = 0.2308$				
M	Distortions	Efficient TFP	Distorted TFP	TFP ^{gain}
1112	$\tau^k, \tau^l, \tau^n \neq 0$	181.45	59.94	202.73%
1112	$\tau^k \neq 0; \tau^l, \tau^n = 0$	181.45	82.79	119.16%
1112	$\tau^l \neq 0; \tau^k, \tau^n = 0$	181.45	164.98	9.98%
1112	$\tau^n \neq 0; \tau^k, \tau^l = 0$	181.45	147.27	23.21%

When I consider net output with only two factors, land and labor, the number of observations increase from 1112 to 4153. The TFP gain (Table 2.9) is small in the presence of land and labor distortions.

Table 2.9: TFP Gains with 2 Factors				
$\gamma = 0.42, \beta = 0.4286$				
M	Distortions	Efficient TFP	Distorted TFP	TFP ^{gain}
4153	$\tau^l, \tau^n \neq 0$	110.31	86.99	26.80%
4153	$\tau^l \neq 0, \tau^n = 0$	110.31	103.09	7.00%
4153	$\tau^l = 0, \tau^n \neq 0$	110.31	91.47	20.59%

It's clear from the estimations in Table 2.8 and 2.9 that land and labor are not as misallocated as capital.

2.7 Conclusion

There are many papers that have studied how misallocated resources are responsible for reduced output and aggregate TFP. I have used microdata on farms to investigate the possible role of misallocation in the agricultural sector of Bangladesh. I have developed a span-of-control model where agricultural production units face four types of distortions that farm operators implicitly take into account during resource allocations. Since these distortions are idiosyncratic, average products vary by farm productivity. These misallocations have substantial effects on aggregate gross (and net) output and TFP. The central finding of this chapter is that, in Bangladesh low agricultural output

and productivity is due to misallocation of intermediate inputs and capital. Although Bangladesh is overpopulated and severely land-constrained, the measured TFP suggests that misallocation of land (and labor) is small.

Although borrowing from formal financial institutions requires collateral, farms face less distortions and pay significantly lower interest rates when they borrow from formal financial institutions (say, banks). The share of loans that farms access from the formal financial lenders is much less than informal and micro-credit lenders, suggesting that both TFP and output could increase if farms had easier and greater access to credit from formal lenders and faced lower capital distortions. Not only should there be greater presence of banks in agriculture (instead of micro-credit institutions) credit taking procedures should be reduced and simplified so that farmers feel more connected with lenders. Information about the prices of intermediate inputs and their availability should be common knowledge so that their distribution becomes more efficient and farmers of all classes (particularly the landless, marginal and small) faced smaller transaction costs. In summary, timely availability of adequate credit is what is most needed to boost output and TFP of Bangladesh agriculture. For future research I would like work with more comprehensive data and adjust the factors for quality. For instance, crops are very sensitive to rainfall and humidity, something I do not take into account in my experiments.

3 Chapter Three

Misallocation in Manufacturing: Firm-level Data from Bangladesh

3.1 Introduction

There are two key stylized facts in the development literature: First, there are large cross country differences in labor productivity (or income per capita) (Hsieh and Klenow, 2010); for instance, there is a 4-fold difference in labor productivity in non-agriculture between the richest and poorest 10% of countries (Caselli, 2005). Second, the bulk of the cross country differences in income per worker are accounted for by differences in TFP (Adamopoulos, 2011). Since income differences are highly correlated with total factor productivity (TFP), one important question is what causes TFP to vary across countries? Misallocation of resources across heterogeneous production units is one key channel that has received considerable attention in the recent macro literature. Among many things that can causes misallocation one is the heterogeneous effects of institutional policies on firms' allocation of resources. There is strong empirical evidence of heterogeneity in TFP across firms in both developed and developing countries within narrowly defined industries (Bartelsman and Doms, 2000; Tybout, 2000). While the direct approach looks at specific policies, the indirect approach looks at the net effect of the underlying policies on the misallocation of factors. The net effect can be gauged from a firm's optimization problem.

In the recent literature misallocation is captured by heterogenous taxes across firms. Notably, Restuccia and Rogerson (2008) consider a model of heterogeneous firms producing a homogeneous good with decreasing returns to technology. They show that when factor prices vary across firms due to idiosyncratic output distortions (output tax, which can be either positive or negative), there

are TFP losses. When output distortions are negatively correlated with firm-level productivity meaning that the more productive firms are subject to large distortions, TFP losses are larger than when distortions are random.

Hsieh and Klenow (2009) build on the theoretical findings of Restuccia and Rogerson (2008) and provide empirical evidence of the affects of misallocation on TFP. They use micro data on manufacturing plants in China, India and the US to measure firm specific wedges, as deviations of marginal revenue products across firms. Firms produce differentiated products using constant returns to technology and are subject to output and capital distortions and show that if the extent of misallocation (variance of tax rates) were equated to the US level, then manufacturing TFP in China would increase by 30 to 50%, and 40 to 60% in India. Taking into account that misspecification and measurement error might overstate the extent of misallocation they also include the estimates of the US. The take-away message is that due to firm-level idiosyncratic distortions firms face different input prices that result in resources being sub-optimally used.

In this chapter, I introduce idiosyncratic distortions and provide quantitative evidence on the impact of resource misallocation on production efficiency in the manufacturing sector of Bangladesh. I follow a model similar to that of Hsieh and Klenow (HK, 2009), but differs along the following dimensions: First, while HK use the Melitz framework of monopolistic competition with CES preference and constant returns to scale technology, I use the Lucas Span-of-Control framework of perfect competition with decreasing returns-to-scale technology.³⁶ Second, I allow for two distortions: capital and labor that drive wedges between the marginal products across firms; HK allow for output and capital distortions. Third, while HK focus on aggregate manufacturing sector, I compare productivity differences and TFP gains across different industries within manufacturing.

³⁶I have diminishing returns in production instead of utility. The two approaches are isomorphic for aggregate TFP.

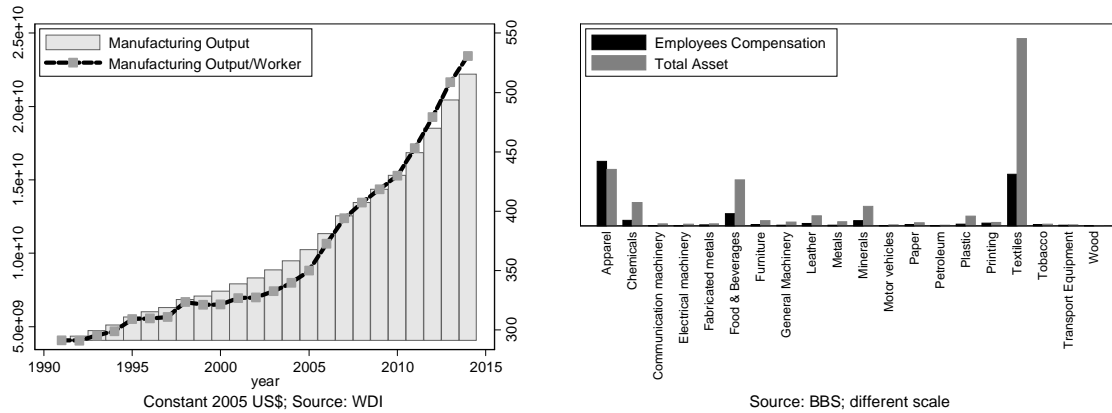
I use firm level micro data from the Bangladesh Industry Data (2005-2006) to measure dispersions in the marginal products of capital and labor within four-digit manufacturing industries. To do that I estimate the distortions that firms face from the residuals in the marginal value products of capital and labor. Then I estimate the potential gain in TFP by hypothetically reallocating resources across firms within each industry. For my benchmark set of parameters and firms facing distortions, I find that if distortions are removed manufacturing TFP can increase between 100 and 500%. In addition I find that capital is more misallocated than labor.

The chapter proceeds as follows. In Section 3.2, I describe specific policies and institutions about Bangladesh that serve to distort the allocation of factors across firms. In section 3.3, I develop a baseline model of perfect competition with heterogeneous firms and derive a measure of aggregate TFP. In section 3.4, I describe in detail the dataset and how the distortions are measured. In section 3.5, I provide evidence on misallocation based on the model developed in section 3.3 and estimate the TFP gains by removing distortions. Section 3.6 has few robustness results and section 3.7 concludes.

3.2 Motivation

There are many institutional factors and policies in Bangladesh that serve to distort the allocation of resources across firms. But the question is, could Bangladesh have done better with its existing resources? Although agriculture is still very important for Bangladesh, the country has made large strides in manufacturing output, dominated by labor intensive industries. Figure 3.1 shows that between 1990 and 2014 while manufacturing output in real value increased by a factor of 5.5 fold manufacturing output per worker increased by less than 2-fold.

Figure 3.1: Manufacturing Output and Industry Shares



Until the mid 80s a sizeable fraction of formal manufacturing was done by state owned firms. These enterprises not only had redundant employment (because of strong labor unions) and access to cheap credit (from state owned financial institutions), but they also enjoyed tax breaks, protection against competition, received subsidies, and could write-off losses year after year for decades. Formal private entrepreneurship was small as it was not profitable. Starting in the mid 80s, many of the state owned firms were liquidated into the hands of politically connected private investors, and the private sector took off, particularly in textile and its vertical industries. Private financial institutions, such as banks and credit unions, were allowed to operate to lend to the manufacturing sector.

Although some level of state ownership in manufacturing still exists, private entrepreneurship has not been smooth. Financial constraints, industrial policies and institutions continue to give rise to heterogeneity in treatment across firms. According to Transparency International, in 2005's corruption index, Bangladesh ranked first, making it the most corrupt country in the world. And

it has marginally improved since then. Every government's department, that entrepreneurs have to deal with from the start to the end, has to be bribed to get business procedures done. The speed at which business steps get done depends on the amount of bribery. Fernandes (2008) empirically shows that more productive firms in Bangladesh are targeted by government officials for bribes that firms end up paying.³⁷ In 2015, Bangladesh positioned 174 among 189 countries in the ranking of 'Doing Business' by the World Bank; in other words, it is not easy to take on and maintain formal entrepreneurship. There is strong evidence that adverse rules and regulations help to create a vibrant informal sector which is the case for Bangladesh (Friedman et al. 2000). According to Porta and Shleifer (2014) informal sector is about 1/5th as efficient as formal sector. The coexistence of both formal and informal sectors has heterogeneous effects on firms. For instance, informal firms, though very small, do not pay taxes and are not subject to industrial regulations. The coexistence of private and state banks is a source of credit market imperfections. There are many irregularities in loan disbursement of both private and state banks. State banks' interest rates aren't significantly lower than private banks' but there are other features of state banks that create heterogeneity in potential borrowers. According to Adhikary (2006), in 1999 almost 41% of all disbursed loans were non-performing. In 2005, non performing loans were 13.56% but for state owned banks it was 21.35% and for state owned financial institutions it was 34.87%. Non-performing loans of private banks was only 5.62% and even much lower for foreign banks. There are 800,000 cases against loan defaulters pending in court. It is easier for entrepreneurs to run-off with loans from state banks than from private banks because they know that the ministry of finance, that oversees the state banks, writes off their losses. So, the slightly lower interest rate and the high probability of getting away with default attract entrepreneurs. But credit is limited and not everyone is politically

³⁷Trade licence, import and export permits, utility connections, VAT, etc.

connected to access this opportunity. The informal sector has zero access to credit from both private and state banks; instead, at best it resorts to microfinance at very high interest rate.

The cost of intermediate inputs may vary significantly across firms within an industry. For instance, the supply of electricity and gas is erratic; 27.8% of 1442 firms surveyed by the World Bank report electricity as the 2nd biggest obstacle for firms (World Bank, 2013). Interruptions in production are very costly, and even costlier for larger firms. To overcome these constraints, there are some firms that generate their own electricity, but for most of the firms it's not cost-effective to generate electricity on their own. Political instability is another obstacle and has heterogeneous effects on firms; 36.7% of firms report political instability as the biggest obstacle for firms (World Bank, 2013). During political unrest, which Bangladesh experiences often, production plants in cities remain shut whereas in export processing zones remain operative. The lack of infrastructure is one of many constraints that inhibit capacity building. The country's infrastructure has improved but is still inadequate to accommodate a population of 160 million. Since building of infrastructure, trade barriers, industry and energy policies are decided by the central government most often political interests are served over business interests.

To promote manufacturing, firms enjoy tax holidays that vary by industry. The structure of tax holiday creates heterogeneity in firms. For instance, as mentioned in Bangladesh Tax Handbook FY:2013-2014, if a factory that produces textile products is located in non-major cities of the Dhaka and Chittagong divisions (equivalent to provinces), then 100% of its income is tax exempt in the 1st and 2nd year, 60% in the 3rd year; pays full tax from the 6th year. If the same plant is set up in other divisions (Rajshahi, Khulna and Barisal), then 100% of its income is tax exempt in the 1st and 2nd year, 70% in the 3rd year; pays full tax from the 8th year. There are many other irregularities in the tax structure that help some firms (within an industry) to be in advantageous position over others.

Another potential source for heterogeneity is in the labor market. The income tax bracket is different for male and female—females with income below 250 thousand *taka* (local currency) are tax-exempt, whereas for males it is 220 thousand. This structure may encourage firms to pay lower wages to female workers. Wages in the informal sector are lower than in the formal sector because there are no contracts between workers and employers. Formal employers provide workers with greater social protection in terms of sick leaves, paid vacation leaves, bonus, pension, etc. Child labor is prohibited by law but since there is no effective way of determining the age of a worker, child labor is prevalent, especially in the informal sector.

Against this multitude of constraints, several labor-intensive industries have become big and important for the economy. Typically, every industry has a trade association, and the bigger the industry in terms of employment, assets and the number of firms, the stronger is the association that can influence the government. For example, the Bangladesh Garment Manufacturers and Exporters Association (BGMEA), representing more than 5000 apparel manufacturing plants, acts as a pressure group to protect the interests of the Ready-Made Garment (RMG) sector. Rather than the government deciding what the minimum wage should be, it has to accept BGMEA mandated minimum wage. Jim Yardley (2013) of the New York times reports that garment factories enjoy subsidies and tax breaks not offered to other industries, and pay less taxes.³⁸ In the right panel of Figure 3.1 the largest industry is the manufacture of wearing apparels, commonly referred to as RMG followed by the manufacture of textiles. The third and fourth industries (catering primarily to domestic markets) are the manufacture of food products and the manufacture of non-metallic minerals. These four industries employed 88.4% of the entire manufacturing workforce of 37 million in

³⁸The headquarters of the BGMEA is built in the middle of the city on an illegal land and continues to operate against Bangladesh High Court ruling from many years ago.

2005. Starting in the 80s, Bangladesh invested substantially in RMG. To encourage export, many types of incentives were and are still given by the central government and importing countries. For instance, not only are raw materials and machinery import tax-exempt, exporters enjoy tax holidays and receive subsidies (known as GSP).³⁹ In the 90s, to create the vertical industries of RMG, large investments took place in the manufacture of textiles, such as spinning, weaving and dyeing. Today, exporters enjoy zero duty access to Europe, North America and other rich economies. Although Bangladesh's labor productivity in textile-related industries is lower than its competitive neighbors' (China, India Vietnam, and Pakistan), Bangladesh, in just 3 decades, has become the largest apparel exporting country after China (Berg, 2008). Financial (banks with specific services) and non-financial (export processing zones, transport system) institutions facilitate these two industries.⁴⁰ Since the manufacturing of food products and the manufacturing of mineral products are primarily non-exporting industries, there are no foreign competitors as a result of which internal policies specific for these industries may be not inherently as distortionary as those for exporting industries.

Having discussed the underlying institutional factors that may have heterogeneous effects on firms' allocation of capital and labor, I present my model in Section 3.3. Instead of trying to assess the implications of any one policy that could be generating misallocation, my objective here is to provide evidence on the overall extent of misallocation within manufacturing industries and assess the implied TFP losses.

³⁹Exemptions from Customs Duty: Capital machinery; Raw materials of Medicine; Poultry Medicine, Feed & machinery; Defence stores; Chemicals of leather and leather goods; Private power generation unit; Textile raw materials and machinery; Solar power equipment; Relief goods; Goods for blind and physically retarded people; and Import by Embassy and UN. [NBR]

⁴⁰Since 2000 manufacturing goods comprise 90% of total exports of Bangladesh.

3.3 Technology

I assume there is a single final good Y produced by a representative firm in a perfectly competitive final output market. This firm combines the output Y_s of S manufacturing industries using a Cobb-Douglas production technology:

$$Y = \prod_{s=1}^S Y_s^{\theta_s}, \quad \text{where } \sum_{s=1}^S \theta_s = 1. \quad (26)$$

θ_s is each industry's share of output in aggregate manufacturing output. The demand for each industry's output Y_s can be derived by the representative firm's cost minimization problem:

$$P_s Y_s = \theta_s P Y, \quad (27)$$

where P_s is the price of industry output Y_s and $P \equiv \prod_{s=1}^S (P_s / \theta_s)^{\theta_s}$ is the price of the final good, which we take as numeraire; therefore, $P = 1$. Each manufacturing industry's output, Y_s , is a linear aggregate of M_s firms' homogeneous output given by

$$Y_s = \sum_{i=1}^{M_s} Y_{si}, \quad (28)$$

where the subscript in Y_{si} denotes firm i in industry s . Given homogeneity in firm's output within industries, each firm's price is the same as the industry price; that is, $P_{si} = P_s$. Each firm has access to a decreasing returns to technology to produce the homogeneous product:

$$Y_{si} = A_{si}^{1-\gamma} (K_{si}^{\alpha_s} N_{si}^{1-\alpha_s})^{\gamma}, \quad (29)$$

where Y_{si} is the output; $A_{si}^{1-\gamma}$ is its production efficiency (TFP); K_{si} and N_{si} are the amounts of capital and labor input respectively. The production technology exhibits decreasing returns to scale at the firm-level as in the Lucas (1978) Span-of-Control model. The parameter γ (referred to as the Lucas span of control) governs the degree of diminishing returns to scale at the firm level. $\alpha_s\gamma$ and $(1 - \alpha_s)\gamma$ are the shares of capital and labor in total output. While γ is common across all firms and across industries, α_s is common only across all firms in each industry. A firm's decision to use capital and hire labor is also constrained by firm-specific capital $(1 + \tau_{Ksi})$ and labor distortions $(1 + \tau_{Nsi})$. Each firm's profits are given by

$$\pi_{si} = P_{si}Y_{si} - (1 + \tau_{Nsi})wN_{si} - (1 + \tau_{Ksi})rK_{si}, \quad (30)$$

where $P_{si}Y_{si}$ is the net value of a firm's production. The effective factor prices of capital and labor are $(1 + \tau_{Ksi})r$ and $(1 + \tau_{Nsi})w$, resulting in varying factor prices across firms. From the first order conditions for the problem (30) with respect to capital and labor, the demand for capital, labor and capital-labor ratio are:

$$\begin{aligned} K_{si} &= A_{si} \left[\gamma P_{si} \left(\frac{1 - \alpha_s}{w} \right)^{\gamma - \alpha_s \gamma} \left(\frac{\alpha_s}{r} \right)^{1 - \gamma + \alpha_s \gamma} \right]^{\frac{1}{1 - \gamma}} \left[\frac{1}{(1 + \tau_{Ksi})^{1 - \gamma + \alpha_s \gamma} (1 + \tau_{Nsi})^{\gamma - \alpha_s \gamma}} \right]^{\frac{1}{1 - \gamma}}, \\ N_{si} &= A_{si} \left[\gamma P_{si} \left(\frac{1 - \alpha_s}{w} \right)^{1 - \alpha_s \gamma} \left(\frac{\alpha_s}{r} \right)^{\alpha_s \gamma} \right]^{\frac{1}{1 - \gamma}} \left[\frac{1}{(1 + \tau_{Ksi})^{\alpha_s \gamma} (1 + \tau_{Nsi})^{1 - \alpha_s \gamma}} \right]^{\frac{1}{1 - \gamma}}, \\ \frac{K_{si}}{N_{si}} &= \frac{\alpha_s}{1 - \alpha_s} \frac{w (1 + \tau_{Nsi})}{r (1 + \tau_{Ksi})}. \end{aligned}$$

Substituting K_{si} and N_{si} into (29) gives each firm's equilibrium output as

$$Y_{si} = A_{si} \left[\gamma P_{si} \left(\frac{1 - \alpha_s}{w} \right)^{1 - \alpha_s} \left(\frac{\alpha_s}{r} \right)^{\alpha_s} \right]^{\frac{\gamma}{1 - \gamma}} \left[\frac{1}{(1 + \tau_{Ksi})^{\alpha_s} (1 + \tau_{Nsi})^{1 - \alpha_s}} \right]^{\frac{\gamma}{1 - \gamma}}. \quad (31)$$

The production efficiency and distortions determine resource allocations across firms and output. From the first order conditions I measure the marginal revenue products of capital (MRPK) and labor (MRPN). MRPK is proportional to the revenue capital ratio and MRPN to the revenue labor ratio. Differences in marginal revenue products across firms affect resource allocations. Firms equate after-tax MRPK with r and after-tax MRPN with w , and these are equalized across firms.

$$\text{MRPK}_{si} \equiv \gamma \alpha_s \frac{P_s Y_{si}}{K_{si}} = (1 + \tau_{si}^K) r \quad (32)$$

$$\text{MRPN}_{si} \equiv \gamma (1 - \alpha_s) \frac{P_s Y_{si}}{N_{si}} = (1 + \tau_{si}^N) w \quad (33)$$

With common factor prices across firms, if $(1 + \tau_{si}^K) > 1$ (which implies a tax), the before tax marginal revenue product of capital must be lower than in firms that do not face distortions. Similarly, if $(1 + \tau_{si}^K) < 1$ (which implies a subsidy), then the before tax marginal revenue product of capital must be higher than in firms that do not receive subsidy. Consider two firms, i and j in an industry s : if there are no distortions, then the allocation of a factor, capital, for instance, is given by

$$\frac{K_{si}}{K_{sj}} = \frac{A_{si}}{A_{sj}}. \quad (34)$$

The more productive firm uses more capital. With distortions, the relative allocation is given by

$$\frac{K_{si}}{K_{sj}} = \frac{A_{si}}{A_{sj}} \left[\left(\frac{1 + \tau_{sj}^K}{1 + \tau_{si}^K} \right)^{1-\gamma+\alpha_s\gamma} \left(\frac{1 + \tau_{sj}^N}{1 + \tau_{si}^N} \right)^{\gamma-\alpha_s\gamma} \right]^{\frac{1}{1-\gamma}}. \quad (35)$$

The relative allocation depends not only on individual TFPs but also on capital and labor distortions. Distortions cause marginal revenue products of capital and labor to vary across firms, with implications for equilibrium allocations of aggregate resources within industries because the more productive firms do not necessarily use more resources; for instance, a firm, even with low A_s but faced distortions in the form of subsidy, uses proportionately more factors than a firm with high A_s —henceforth, the misallocation, which affects output and TFP of all industries separately within manufacturing. Capital and labor are mobile across firms within each industry; therefore, the resource constraints are

$$\sum_{i=1}^{M_s} K_{si} = K_s, \quad \sum_{i=1}^{M_s} N_{si} = N_s \quad \text{and} \quad \sum_{i=1}^{M_s} Y_{si} = Y_s.$$

To find an expression for the observed manufacturing TFP (define as $\text{TFP}^{\text{distorted}}$) as a function of distortions I need to find the industry-specific TFPs and combine them according to each industry's share in aggregate output. First, I divide each firm's demand for capital and labor by its corresponding industry demand for capital ($\sum K_{si} = K_s$) and labor ($\sum N_{si} = N_s$). Next, I divide both the numerator and denominator (of the middle term in equation (36) and equation (37)) by the relevant factor price which expresses K_{si} and N_{si} as functions of resources available for a given industry and the weighted average of the value of the marginal product of capital ($\overline{\text{MRPK}}$) and labor ($\overline{\text{MRPN}}$).⁴¹

⁴¹ $\overline{\text{MRPK}} = \frac{r}{\sum \frac{py_i}{(1+\tau_i^k)} \frac{1}{pY}}; \overline{\text{MRPN}} = \frac{w}{\sum \frac{py_i}{(1+\tau_i^l)} \frac{1}{pY}}.$

$$K_{si} = \left[\frac{r}{\underbrace{\sum_{i=1}^{M_s} \frac{P_s Y_{si}}{(1+\tau_{Ksi})} \frac{1}{P_s Y_s}}_{\overline{\text{MRPK}}_s}} \frac{\frac{P_s Y_{si}}{(1+\tau_{Ksi})} \frac{1}{P_s Y_s}}{r} \right] K_s = \overline{\text{MRPK}}_s \frac{\frac{P_s Y_{si}}{(1+\tau_{Ksi})} \frac{1}{P_s Y_s}}{r} K_s \quad (36)$$

$$N_{si} = \left[\frac{w}{\underbrace{\sum_{i=1}^{M_s} \frac{P_s Y_{si}}{(1+\tau_{Nsi})} \frac{1}{P_s Y_s}}_{\overline{\text{MRPN}}_s}} \frac{\frac{P_s Y_{si}}{(1+\tau_{Nsi})} \frac{1}{P_s Y_s}}{w} \right] N_s = \overline{\text{MRPN}}_s \frac{\frac{P_s Y_{si}}{(1+\tau_{Nsi})} \frac{1}{P_s Y_s}}{w} N_s \quad (37)$$

Since $\sum_{s=1}^S K_s = K$, $\sum_{s=1}^S N_s = N$ and $\theta_s PY = P_s Y_s$, I combine the industry demands with the allocation of total expenditure across all industries to get each industry's demand for capital in terms of K and its share of the weighted average of $\overline{\text{MRPK}}_s$ in total $\overline{\text{MRPK}}_{s'}$ in equation (38), and each industry's demand for labor in terms of N and its share of the weighted average of $\overline{\text{MRPN}}_s$ in total $\overline{\text{MRPN}}_{s'}$ in equation (39).

$$K_s = K \times \frac{\alpha_s \theta_s / \overline{\text{MRPK}}_s}{\sum_{i=1}^{M_s} \alpha_{s'} \theta_{s'} / \overline{\text{MRPK}}_{s'}} \quad (38)$$

$$N_s = N \times \frac{(1 - \alpha_s) \theta_s / \overline{\text{MRPN}}_s}{\sum_{i=1}^{M_s} (1 - \alpha_{s'}) \theta_{s'} / \overline{\text{MRPN}}_{s'}} \quad (39)$$

By substituting K_{si}, N_{si} into Y_{si} , I find each industry's output as a function of K_s, N_s, M_s (number of firms in industry s) and TFP_s .

$$Y_{si} = \left\{ A_{si} K_s^{\frac{\alpha_s \gamma}{1-\gamma}} N_s^{\frac{(1-\alpha_s) \gamma}{1-\gamma}} (\overline{\text{MRPK}}_s)^{\frac{\alpha_s \gamma}{1-\gamma}} (\overline{\text{MRPN}}_s)^{\frac{(1-\alpha_s) \gamma}{1-\gamma}} \left(\frac{1}{Y_s} \right)^{\frac{\gamma}{1-\gamma}} \right. \\ \left. \left[\frac{1}{r(1 + \tau_{Ksi})} \right]^{\frac{\alpha_s \gamma}{1-\gamma}} \left[\frac{1}{w(1 + \tau_{Nsi})} \right]^{\frac{(1-\alpha_s) \gamma}{1-\gamma}} \right\}$$

and since $Y_s = \sum_{i=1}^M Y_{si}$, outputs of all firms in an industry are linearly aggregated to find an industry's output as

$$Y_s = \left\{ \frac{(\overline{\text{MRPK}}_s)^{\alpha_s \gamma} (\overline{\text{MRPN}}_s)^{(1-\alpha_s) \gamma}}{M_s^{1-\gamma}} \left\{ \sum_{i=1}^{M_s} A_{si} \left[\frac{1}{r(1 + \tau_{Ksi})} \right]^{\frac{\alpha_s \gamma}{1-\gamma}} \left[\frac{1}{w(1 + \tau_{Nsi})} \right]^{\frac{(1-\alpha_s) \gamma}{1-\gamma}} \right\}^{1-\gamma} \right. \\ \left. \cdot \underbrace{(K_s^{\alpha_s} N_s^{1-\alpha_s})^\gamma}_{\text{Industry Resources}} M_s^{1-\gamma} \right\} \quad (40)$$

$$= \text{TFP}_s (K_s^{\alpha_s} N_s^{1-\alpha_s})^\gamma M_s^{1-\gamma} \quad (41)$$

and since $Y = \prod_{s=1}^S Y_s^{\theta_s}$, outputs of all industries are combined to find aggregate manufacturing output as

$$Y = \prod_{s=1}^S (\text{TFP}_s \times (K_s^{\alpha_s} \times N_s^{1-\alpha_s})^\gamma \times M_s^{1-\gamma})^{\theta_s}. \quad (42)$$

To derive an exact expression for firm-level TFP, I adopt two measures of productivity, now commonly used in literature (Forster et al, 2008): The first measure is *physical productivity* (TFPQ) that estimates TFP based on revenue deflated with firm-level prices is defined as

$$\text{TFPQ}_{si} = A_{si}^{1-\gamma} = \frac{Y_{si}}{\left[K_{si}^{\alpha_s} (wN_{si})^{1-\alpha_s} \right]^\gamma}. \quad (43)$$

The survey data do not have firm-level prices; however, homogeneity in firm's output gives $P_s = P_{si}$, so I use industry indices to estimate TFPQ_{si} . The second measure, *revenue productivity* (TFPR), that estimates TFP based on revenue is defined as

$$\text{TFPR}_{si} = \frac{P_{si}Y_{si}}{K_{si}^{\alpha_s} N_{si}^{1-\alpha_s}} = \left(\frac{P_{si}Y_{si}}{K_{si}^{\alpha_s}} \right)^{\alpha_s} \left(\frac{P_{si}Y_{si}}{N_{si}^{1-\alpha_s}} \right)^{1-\alpha_s}. \quad (44)$$

TFPR can be expressed in terms of distortions, and by substituting revenue-capital (32) and revenue-labor (33) ratios into equation (44) it becomes

$$\begin{aligned} \text{TFPR}_{si} &= (1 + \tau_{Ksi})^{\alpha_s} (1 + \tau_{Nsi})^{1-\alpha_s} \left(\frac{r}{\alpha_s \gamma} \right)^{\alpha_s} \left(\frac{w}{(1 - \alpha_s) \gamma} \right)^{1-\alpha_s} \\ &= \left(\frac{\text{MRPK}_{si}}{\alpha_s \gamma} \right)^{\alpha_s} \left(\frac{\text{MRPN}_{si}}{(1 - \alpha_s) \gamma} \right)^{1-\alpha_s}. \end{aligned} \quad (45)$$

TFPR_{si} summarizes firm-specific distortions and is proportional to a geometric average of a firm's MRPK_{si} and MRPN_{si} . If resource allocation is efficient, then TFPR_{si} becomes industry specific and should not vary across firms within an industry. Firms with high TFPQ should receive more resources until all firm's TFPR are equal within an industry. However, if TFPR_{si} varies across firms, then it is due to the underlying systematic and idiosyncratic firm-level distortions that firms face. For instance, if a firm's TFPR_{si} is large, then the constraints it faces alter its effective marginal products, making it smaller than optimally sized, simply because it uses less resources.

Combining the expressions for TFPR_{si} and TFPQ_{si} the estimated measure of each industry's

TFP_s is

$$\text{TFP}_s^{\text{distorted}} = \left\{ \frac{\sum_{i=1}^{M_s} \left[\text{TFPQ}_{si} \left(\frac{\overline{\text{TFPR}}_s}{\text{TFPR}_{si}} \right)^\gamma \right]^{\frac{1}{1-\gamma}}}{M_s} \right\}^{1-\gamma}, \quad (46)$$

where $\overline{\text{TFPR}}_s$ is a geometric average of the average marginal revenue product of capital and labor.⁴²

Equation (46) is the key equation I use for my empirical estimates. If the gap between $\overline{\text{TFPR}}_s$ and TFPR_{si} widens, then the difference between them would weigh into TFP adversely. If there were no distortions and marginal products were equalized across firms, then $\overline{\text{TFPR}}_s = \text{TFPR}_{si}$, in which case each industry's measure of TFP is efficient and given by

$$\text{TFP}_s^{\text{efficient}} = \left\{ \frac{\sum_{i=1}^{M_s} [\text{TFPQ}_{si}]^{\frac{1}{1-\gamma}}}{M_s} \right\}^{1-\gamma}, \quad (47)$$

and firms allocate resources according to their true productivity TFPQ. The ratio of the distorted and efficient TFP_s gives a measure of industry-specific TFP gain given by

$$\text{TFP}_s^{\text{gain}} = \left\{ \frac{\left\{ \sum_{i=1}^{M_s} [\text{TFPQ}_{si}]^{\frac{1}{1-\gamma}} \right\}^{1-\gamma}}{\left\{ \sum_{i=1}^{M_s} \left[\text{TFPQ}_{si} \left(\frac{\overline{\text{TFPR}}_s}{\text{TFPR}_{si}} \right)^\gamma \right]^{\frac{1}{1-\gamma}} \right\}^{1-\gamma}} \right\}. \quad (48)$$

The efficient TFP_s (in terms of TFPQ_{si}) and distorted TFP_s (in terms of TFPQ_{si} , TFPR_{si} and $\overline{\text{TFPR}}_s$) can be separately combined into aggregate output (equation ((42)) to find an exact expres-

⁴² $\overline{\text{TFPR}}_s = \left(\frac{\overline{\text{MRPK}}_{si}}{\alpha_s \gamma} \right)^{\alpha_s} \left(\frac{\overline{\text{MRPN}}_{si}}{(1-\alpha_s)\gamma} \right)^{1-\alpha_s}$
 $= \left(\frac{r}{\gamma \alpha_s} \right)^{\alpha_s} \left(\frac{w}{\gamma(1-\alpha_s)} \right)^{(1-\alpha_s)} \frac{1}{\left[\sum \frac{P_s Y_{si}}{(1+\tau_{Ksi}) P_s Y_s} \right]^{\alpha_s} \left[\sum \frac{P_s Y_{si}}{(1+\tau_{Nsi}) P_s Y_s} \right]^{(1-\alpha_s)}}$

sion for the efficient and distorted manufacturing output given by

$$Y^{\text{efficient}} = \prod_{s=1}^S \left\{ \left(K_s^{\alpha_s} \times N_s^{1-\alpha_s} \right)^\gamma \times M_s^{1-\gamma} \left\{ \frac{\sum_{i=1}^{M_s} [\text{TFPQ}_{si}]^{\frac{1}{1-\gamma}}}{M_s} \right\}^{1-\gamma} \right\}^{\theta_s}, \quad (49)$$

$$Y^{\text{observed}} = \prod_{s=1}^S \left\{ \left(K_s^{\alpha_s} \times N_s^{1-\alpha_s} \right)^\gamma \times M_s^{1-\gamma} \left\{ \frac{\sum_{i=1}^{M_s} \left[\text{TFPQ}_{si} \left(\frac{\text{TFPR}_s}{\text{TFPR}_{si}} \right)^\gamma \right]^{\frac{1}{1-\gamma}}}{M_s} \right\}^{1-\gamma} \right\}^{\theta_s} \quad (50)$$

$Y^{\text{efficient}}$ is the potential manufacturing output of Bangladesh when there are no distortions at industry-level and marginal revenue products are equal across firms within an industry for all industries. Y^{observed} is the reduced out with firm-level distortions. Finally, the ratio of (49) and (50) gives aggregate manufacturing TFP-gain.

$$\text{TFP}_{\text{manufacturing}}^{\text{gain}} = \frac{\prod_{s=1}^S \left\{ \sum_{i=1}^{M_s} [\text{TFPQ}_{si}]^{\frac{1}{1-\gamma}} \right\}^{(1-\gamma)\theta_s}}{\prod_{s=1}^S \left\{ \sum_{i=1}^{M_s} \left[\text{TFPQ}_{si} \left(\frac{\text{TFPR}_s}{\text{TFPR}_{si}} \right)^\gamma \right]^{\frac{1}{1-\gamma}} \right\}^{(1-\gamma)\theta_s}}. \quad (51)$$

3.4 Data Set for Bangladesh Manufacturing and Distortions

My data are from the Survey of Manufacturing Industries (SMI) 2005, conducted by government regulated Bangladesh Bureau of Statistics (BBS). Every 5 years, a survey of the registered manufacturing plants is conducted that provides important industry-specific information on industrial structure, ownership status, employment, intermediate consumption (inputs), value of fixed assets, gross output and gross value added. In 2005, 34710 firms of various sizes (by employment) cover-

ing 23 industries (at 2-digit ISIC 15 to 37) were surveyed. 98.41% of the surveyed firms are under private ownership and the rest under government or joint ownership. Information is available for 6064 firms that employ more than 10 workers.⁴³ Accounting for only firms with positive output, I exclude ISIC 30, 33 and 37 because producer price indices are not available for these industries. Firms that have missing data on intermediate inputs, fixed assets or wage are also excluded.

To estimate firm-level $TFPQ_{si}$ I need value-added output (Y_{si}), capital (K_{si}) and labor (N_{si}). First, I subtract the the total value of raw materials and intermediate inputs (reported as industrial costs, non-industrial costs and indirect taxes) from a firm's gross output $P_s Y_{si}$. Since a firm belongs to a particular industry under ISIC and homogeneity guarantees $P_{si} = P_s$, I deflate its revenue $P_{si} Y_{si}$ by P_s (industry price index reported by BBS.) to find real output (Y_{si}). Aggregate valued-added manufacturing output is $\sum_{s=1}^S \sum_{i=1}^{M_s} Y_{si}$, where S is the number of industries and M_s is the number of firms in each industry s .⁴⁴ From equation (49) $Y^{\text{observed}} = \prod_{s=1}^S \left(TFP_s^{\text{distorted}} (K^{\alpha_s} N_s^{1-\alpha_s})^\gamma M_s^{1-\gamma} \right)^{\theta_s}$. Every firm reports total wages (wN_{si}) paid to its workers (that includes all fringe benefits) and its net capital stock (K_{si}) that includes building, machinery and transport equipment. Therefore, an industry's labor and capital are $wN_s = \sum_i^{M_s} wN_{si}$ and $K_s = \sum_i^{M_s} K_{si}$; and for the whole economy aggregate labor and capital are $\sum_{s=1}^S \sum_{i=1}^{M_s} wN_{si}$ and $\sum_{s=1}^S \sum_{i=1}^{M_s} K_{si}$. Firm-level K_{si} and wN_{si} are deflated by 2010 price of capital and CPI (both from the PWT 8.1). I do not take into account differences in the quality of workers employed and assets used by firms.

The distortions can be isolated from the marginal revenue products. From the marginal revenue product of capital (32)

⁴³Firms that employ between 1 and 10 workers are considered Small and Medium Enterprises (SME). Data for SME are not available.

⁴⁴Total number of firms is $M = \sum_{s=1}^S M_s$.

$$(1 + \tau_{si}^K) = \frac{\gamma \alpha_s}{r} \frac{P_s Y_{si}}{K_{si}}. \quad (52)$$

From the marginal revenue product of labor (33)

$$(1 + \tau_{si}^N) = \frac{\gamma(1 - \alpha_s)}{w} \frac{P_s Y_{si}}{N_{si}}. \quad (53)$$

It does not matter what values of r and w are chosen for firm-level distortions because, in the estimation of TFPs, they get cancelled out in the ratio of $\overline{\text{TFPR}}_s$ and TFPR_{si} .⁴⁵ Common to all firms is the span of control parameter (γ); the higher the span the greater are the distortions (equations 52 and 53). Hsieh and Klenow (2009)'s implied value for γ is a conservative 0.50 (which is equivalent to $\sigma = 3$, as the elasticity of substitution in their monopolistic framework). Therefore, for my benchmark model I choose $\gamma = 0.50$ (used for the derivation of firm-level TFPQ for the figures in the next section). For comparability, considering that Atkeson and Kehoe (2005) chose $\gamma \geq 0.80$ based on diminishing returns in production and utility, I also do the estimations using $\gamma = 0.60, 0.70$ and 0.80 . The elasticity of output with respect to capital (α_s) is not set on the basis of 1 minus labor shares in Bangladesh manufacturing. Since I cannot separate labor and capital distortions from actual labor and capital elasticities, for each industry I use the labor share in the corresponding industry in the US manufacturing. The US shares are relatively undistorted across firms and industries, and are commonly used for empirical estimations as found in the literature. The industry-specific α_s , therefore, is 1 minus the estimated average (over 1998-2011) of compensation of employees in value added for each U.S. manufacturing industry, listed in Table B1 in the

⁴⁵For firm level distortion I choose $r, w = 1$.

Appendix.⁴⁶ Finally, to make estimations robust to outliers I trim the 1% tails of firm specific distortions ($TFPR_{si}$) which reduces the number of observations to 5267 firms. Table 3.1 lists the number of firms and share of valued added in aggregate manufacturing of each industry.

Table 3.1: Number of Firms and Size by Industry (Source: SMI 2005-06, BBS)

ISIC	Industry Classification	Pre-trimming		Post-trimming	
		M_s	θ_s	M_s	$\theta_{s,\gamma=0.50}$
15	Manu. of Food Products and Beverages	1306	0.1400	1278	0.1215
16	Manu. of Tobacco Products	50	0.0083	48	0.0088
17	Manu. of Textiles	1542	0.4077	1510	0.4294
18	Manu. of Wearing Apparels	678	0.1995	664	0.2037
19	Manu. of Leather and Related Products	111	0.0252	107	0.0187
20	Manu. of Wood and Related Products	116	0.0031	112	0.0035
21	Manu. of Paper and Paper Products	63	0.0124	61	0.0126
22	Publishing, Printing, Reproduction of Recorded Media	171	0.0143	167	0.0149
23	Manu. of Coke and Refined Petroleum Products	13	0.0017	11	0.0014
24	Manu. of Chemicals and Chemical Products	147	0.0657	143	0.0727
25	Manu. of Rubber and Plastic Products	131	0.0190	127	0.0212
26	Manu. of Other Non-Metallic Mineral Products	508	0.0429	496	0.0346
27	Manu. of Basic Metals	62	0.0164	60	0.0185
28	Manu. of Fabricated Metal Products	145	0.0137	141	0.0134
29	Manu. of Machinery and Equipment	51	0.0064	49	0.0051
31	Manu. of Electrical Equipment	21	0.0050	19	0.0055
32	Manu. of Radio, Television & Communication Equipment	5	0.0053	3	0.0013
34	Manu. of Motor Vehicles, Trailers and Semi-trailers	8	0.0027	6	0.0015
35	Manu. of Transport Equipment	54	0.0045	52	0.0051
36	Manu. of Furniture	219	0.0062	213	0.0064

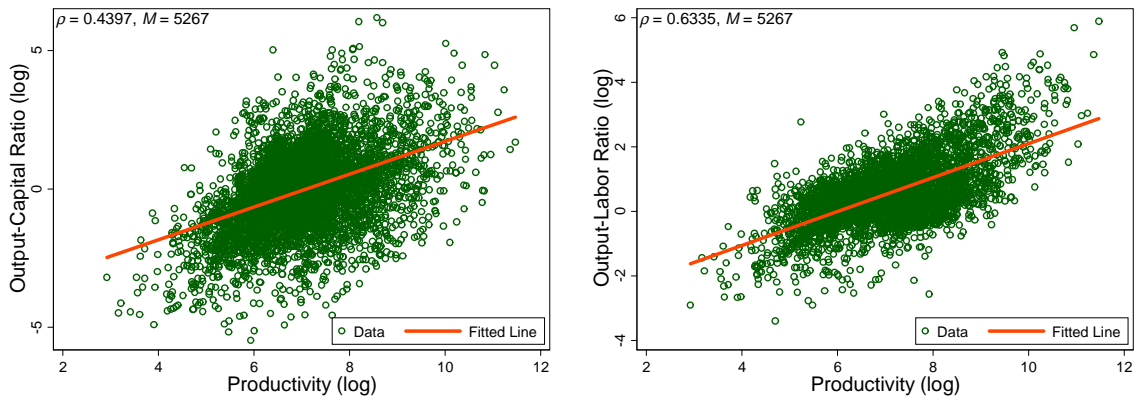
3.5 Evidence of Misallocation and Gains from Reallocation

For a given technology, the efficient allocation of factors is such that the more productive firms should operate with more capital and labor and produce more output. In other words, at an aggregate level the correlation between any factor and productivity should be significantly positive. In standard models like Lucas (1978) and Hopenhayn (1992), in the absence of distortions, marginal products of factors should be equalized across firms which implies equalization of marginal products. In other words, if there is no misallocation at an aggregate level, then average products should not vary by TFP. Higher production units should produce more and command more factors.

⁴⁶Industry Economic Accounts Directorate, Bureau of Economic Analysis, U.S. Department of Commerce.

Using the measure of productivity (as in equation 43), simple plots in Figure 3.2 between average product of capital and TFPQ and between average product labor and TFPQ can reveal the extent of misallocation in manufacturing.⁴⁷

Figure 3.2: Average Product by Productivity



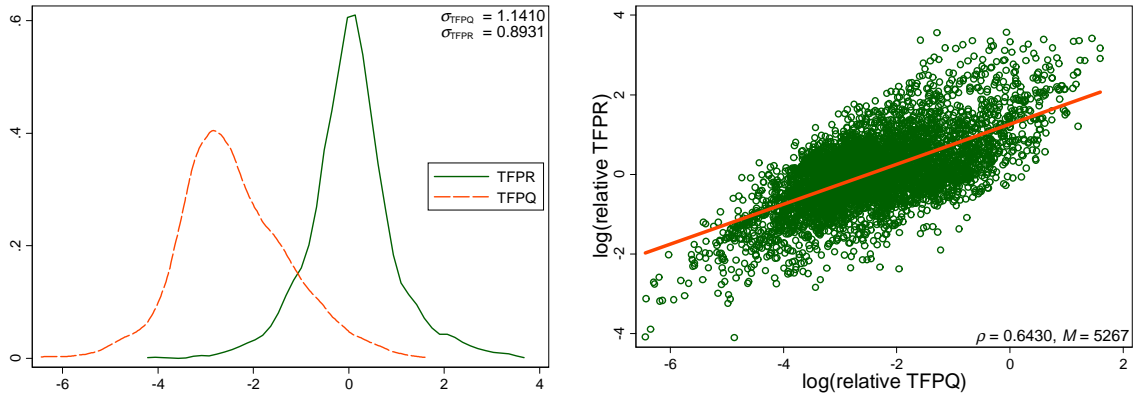
In the two panels of Figure 3.2 the correlation between marginal product and productivity is positive and significantly different from zero; in other words, implied marginal product of capital and labor are not equated across firms indicating misallocation. If marginal products were the same across all firms there would be no significant correlation between marginal products and firm-level TFPs. Instead, the correlations are 0.4397 and 0.6335, indicating that the actual allocations of capital and labor across firms in Bangladesh manufacturing are unrelated to productivity. Figure 3.3, more than figure 3.2, shows stronger evidence of capital and labor misallocation across firms.

In the left panel of Figure 3.3, $TFPQ_{si}$ is noticeably more dispersed than $TFPR_{si}$ (both relative to

⁴⁷Given Cobb-Douglas production function, $APK = \frac{MPK}{\alpha_s \gamma}$ and $APN = \frac{MPN}{(1-\alpha_s)\gamma}$. $(1 - \alpha_s)$ are the mean values in Table B1 in the Appendix. $\gamma = 0.50$.

industry means weighted by their value added shares in each industry). TFPQ and TFPR are highly correlated with a coefficient of 0.6430, shown in the right panel; in other words, the more productive firms are subject to higher distortions that adversely affects aggregate TFP. The covariance between TFPR and TFPQ (both relative to industry means) is 0.6552.

Figure 3.3: Distribution of Manufacturing TFPQ and TFPR



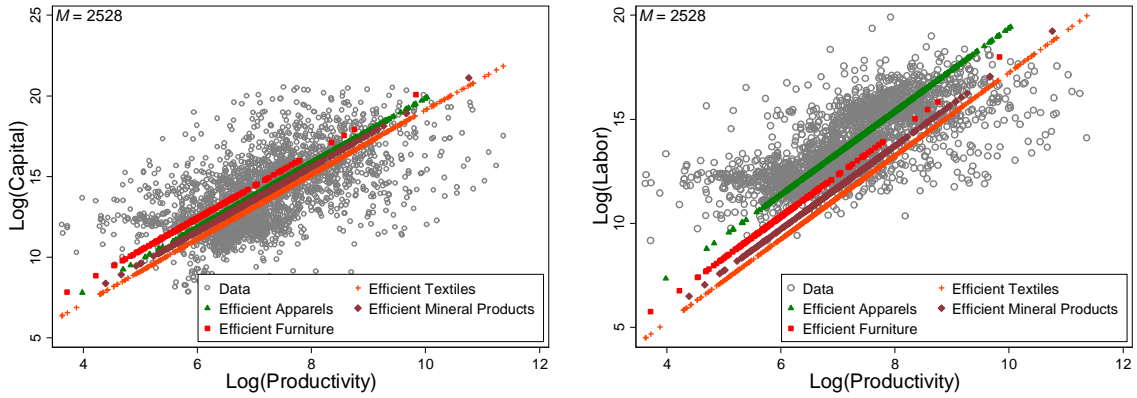
Hsieh and Klenow (2009) provide TFPR and TFPQ dispersion statistics for China, India and the US. In 2005, the standard deviation of TFPQ is 0.95 in China, 1.23 in India and 0.84 in the US; the standard deviation of TFPR is 0.63 in China, 0.67 in India and 0.49 in the US. In Bangladesh the standard deviation of TFPQ is 1.14, which is between China and India, but the standard deviation of TFPR is much higher than that of India.

If there are no distortions ($\overline{\text{TFPR}}_s = \text{TFPR}_{si}$), then each firm's share of capital (labor), within an industry, depends on aggregate capital (labor) and its weighted TFPQ_{si} given by

$$K_{si} = \frac{\text{TFPQ}_{si}^{\frac{1}{1-\gamma}}}{\sum_{i=1}^{M_s} \text{TFPQ}_{si}^{\frac{1}{1-\gamma}}} K_s \quad \text{and} \quad N_{si} = \frac{\text{TFPQ}_{si}^{\frac{1}{1-\gamma}}}{\sum_{i=1}^{M_s} \text{TFPQ}_{si}^{\frac{1}{1-\gamma}}} w N_s.$$

In Figure 3.4, I plot the efficient allocations of capital and labor for four industries (Textiles, Apparels, Mineral Products and Furniture) against productivity. Although the levels of distortions are dissimilar across industries, when efficiently allocated the more productive firms receive more of capital and labor. With distortions, firms use resources disproportionately to productivity.

Figure 3.4: Efficient Allocation by Productivity



Next, following (48), if marginal products were equalized across firms in an industry and resources were hypothetically reallocated, then TFP gain in each industry increases. Table 3.2 lists TFP gains of all industries for $\gamma \in \{0.50, 0.60, 0.70, 0.80\}$.

Table 3.2: TFP gains by Industry and Span-of-Control

Manufacture of	M_s	Efficient/Distorted TFP			
		$\gamma = 0.50$	$\gamma = 0.60$	$\gamma = 0.70$	$\gamma = 0.80$
Food, Beverage	1278	2.16	2.94	4.42	7.46
Tobacco	48	2.59	3.31	4.35	5.94
Textiles	1510	2.06	2.82	4.26	7.28
Apparels	664	1.46	1.73	2.24	3.28
Leather	107	1.87	2.37	3.13	4.26
Wood	112	1.51	1.74	2.04	2.45
Paper	61	1.97	2.75	4.20	6.71
Printing	167	2.07	3.04	4.92	8.48
Petroleum	11	1.75	2.21	2.93	3.96
Chemicals	143	2.81	4.02	6.19	10.08
Plastic	127	1.98	2.56	3.54	5.13
Non-metal Minerals	496	2.10	2.88	4.30	6.84
Metals	60	1.96	2.46	3.24	4.46
Fabricated Metals	141	1.96	2.53	3.48	5.15
General Machinery	49	1.35	1.51	1.74	2.13
Electrical Machinery	19	1.49	1.67	1.90	2.22
Communication Machinery	3	1.11	1.14	1.17	1.21
Motor Vehicles	6	1.07	1.11	1.18	1.34
Transport Equipment	52	1.11	1.15	1.21	1.29
Furniture	213	1.75	2.12	2.71	3.95
Manufacturing Aggregate	5267	1.95	2.57	3.71	5.93

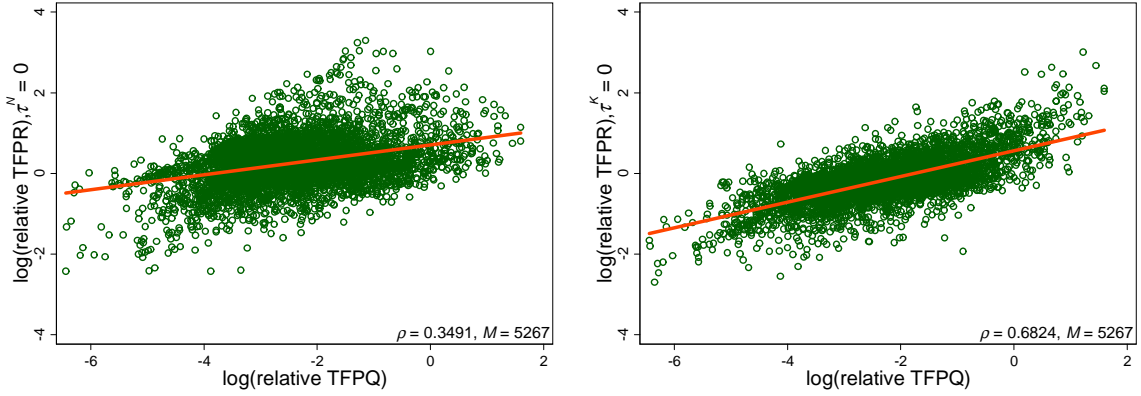
If all firms in an industry faced the same level of distortions, $TFPR_s = TFPR_{si}$ and TFP_s would be efficient. However, there is variation in $TFPR_{si}$ which gives rise to distorted TFP_s . Productivity of manufacturing industries can be compared by looking at industry-level TFP gains. The TFP gains are increasing in the span-of-control (γ) parameter. Although γ increases firm-level distortions, it does not affect the summary measure of relative distortions ($TFPR_s/TFPR_{si}$); instead it affects TFP_s via two channels. First, since $TFPQ_{si}$ is measured as residuals from the firm-level production functions, γ directly reduces variation in $TFPQ_{si}$. Second, since $TFPR_s/TFPR_{si}$ has non-linear effect on TFP_s the higher the value of γ the greater is the effect on TFP_s . For every value of γ , depending on the number of firms within a narrowly defined industry, the TFP gains vary across industries. With some exceptions, the higher the number of firms in an industry the greater seems to be the extent of resource misallocation, implied by the potential TFP_s gains in Table 3.2. For instance, Manufacture of Textiles has 1278 firms and TFP gain is more than 7-

fold (628%) when $\gamma = 0.80$ and more than 2-fold (106%) when $\gamma = 0.50$. Not surprisingly, the TFP gain is relatively small when there are few firms in an industry. The Manufacture of Communication Machinery, with only 3 firms, shows a TFP-gain of 11% when $\gamma = 0.50$. The TFP gain of Manufacture of Apparels, a large industry, is more than 3-fold (228%) when $\gamma = 0.80$. Between the two complementary industries, the Apparels is significantly more efficient than Textiles, reflecting the strength of its entrepreneurs and favorable policies undertaken by different regimes to promote this industry. The TFP-gain of Manufacture of Non-metallic Minerals, also a large industry, is more than 6-fold when $\gamma = 0.80$. The Manufacture of Food and Beverage is also inefficient as its TFP gain is more than 7-fold when $\gamma = 0.80$. To find the weighted TFP gain of manufacturing I use equation (51) where the TFP gain of each industry is combined according to its share of output in manufacturing. If $\gamma = 0.50$ the TFP gain is less than 2-fold (95%), which falls within Hsieh and Klenow's (2009) findings of 47%-127% TFP gain for China, India and the US for 2005. With higher γ TFP gains are considerably larger from equalizing TFPR levels because TFPR gaps reduces more slowly in response to reallocation of inputs from low to high TFPR plants. Finally, for $\gamma = 0.50$ the output loss, which is the ratio of actual (inefficient) manufacturing output $\sum_{s=1}^S \sum_{i=1}^{M_s} Y_{si}$ to its efficient counterpart $Y = \prod_{s=1}^S \left(\text{TFP}_s^{\text{efficient}} (K_s^{\alpha_s} N_s^{1-\alpha_s})^\gamma M_s^{1-\gamma} \right)^{\theta_s}$, is 0.51. In other words, due to resource misallocation only 51% of potential output is produced with existing resources available in each industry.

Next, I look into which type of distortion, capital or labor, contributes the most to TFP losses. If TFPR_{si} is strongly correlated with TFPQ_{si} , then more productive units face higher distortions. In Figure 3.5 the correlation of 0.3491 between TFPR_{si} (with capital distortions, $\tau^N = 0$) and TFPQ_{si} and the correlation of 0.6824 between TFPR_{si} (with labor distortions, $\tau^K = 0$) and TFPQ_{si} suggest

that more productive firms face higher labor distortion than capital distortion.⁴⁸

Figure 3.5: Capital and Labour Distortions by Productivity



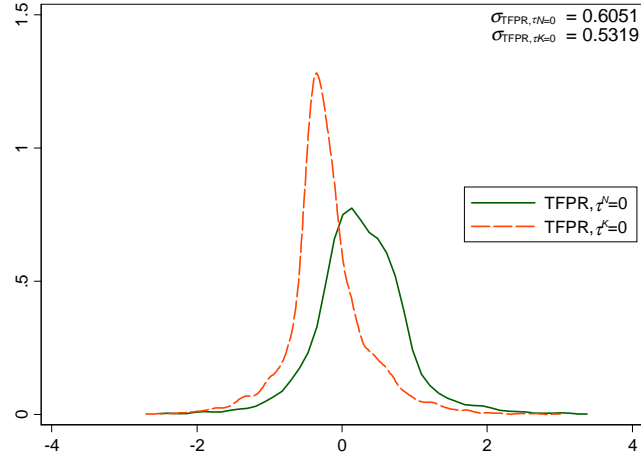
But the average level of distortions is less important than variation in idiosyncratic distortions (Bond et al, 2013), so the larger the dispersion of TFPR_{si} the more the misallocation.⁴⁹ The standard deviations in Figure 3.6 suggest that there is greater dispersion in relative TFPR_{si} (with only capital distortions) than in relative TFPR_{si} (with only labor distortions); therefore, capital distortions have greater effects on TFP.

⁴⁸ $\log(\text{TFPR}_{si, \tau^N=0}) = \frac{(1+\tau_{si}^K)^\alpha}{\left(\sum \frac{1}{1+\tau_{si}^K} \frac{P_{si} Y_{si}}{P_s Y_s}\right)^\alpha}$ and $\log(\text{TFPR}_{si, \tau^K=0}) = \frac{(1+\tau_{si}^N)^{1-\alpha}}{\left(\sum \frac{1}{1+\tau_{si}^N} \frac{P_{si} Y_{si}}{P_s Y_s}\right)^{1-\alpha}}$. TFPR_{si} and TFPQ_{si}

are all relative to weighted industry average.

⁴⁹ In absence of distortions $\text{TFPR}_{si} = 1$.

Figure 3.6: Distribution of Capital and Labour Distortions



I do not isolate the partial effect of each type of distortion in the presence of both distortions; instead, I assume firms are subject to either capital or labor distortion and then re-estimate the TFP gains. So, if capital distortions were removed in the presence of capital distortions only (zero labor distortions), then, at an aggregate level, the TFP gains would be larger than what it would be if labor distortions were removed in the presence of labor distortions only (zero capital distortions). To find the contribution of capital (labor) distortion separately I set firm-level labor (capital) distortion to zero keeping capital (labor) distortion to its actual level. The contribution of capital (labor) distortion is shown in column 3 (column 4) of Table 3.3, which is the percentage gain between the efficient and distorted TFP.

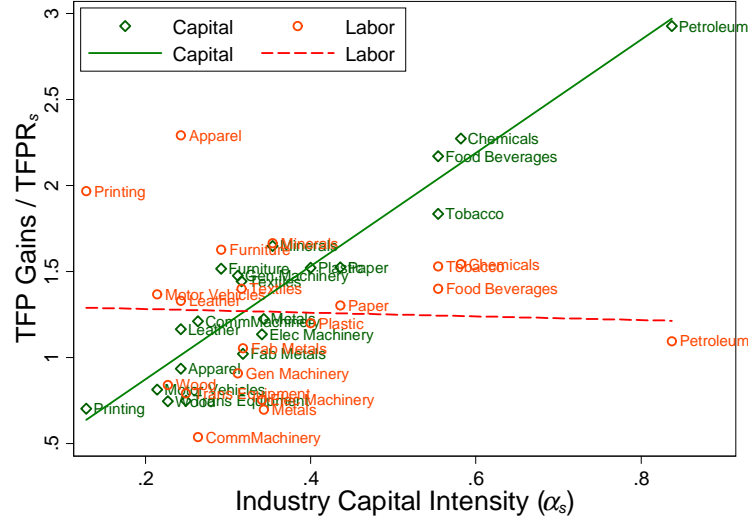
Table 3.3: TFP gains with Capital, Labor Distortions

$(\gamma = 0.50)$	% gain between distorted and efficient TFP with		
	$\tau_{si}^K \neq 0$	$\tau_{si}^K \neq 0$	$\tau_{si}^K = 0$
	$\tau_{si}^N \neq 0$	$\tau_{si}^N = 0$	$\tau_{si}^N \neq 0$
Food and Beverage	116.37	59.71	41.46
Tobacco	158.90	101.66	29.73
Textiles	106.43	40.12	53.10
Apparels	46.07	16.48	29.40
Leather	86.70	24.45	56.67
Wood	51.46	19.92	31.08
Paper	96.60	60.27	46.53
Printing	107.32	12.07	96.86
Petroleum	74.78	66.05	8.82
Chemicals	181.17	68.50	64.16
Plastic	97.72	27.27	60.84
Minerals	110.05	39.14	53.11
Non-metal Minerals	96.16	41.86	50.45
Fabricated Metals	95.95	41.84	54.58
General Machinery	35.05	24.35	11.12
Electrical Machinery	48.83	22.30	21.10
Communication Machinery	11.05	8.64	0.25
Motor Vehicles	7.23	1.57	6.13
Transportation Equipment	11.31	5.09	6.56
Furniture	74.83	30.25	40.35

If capital distortions are removed, then one should expect large TFP gains in capital intensive industries; and if labor distortions are removed, then one should expect large TFP gains in labor intensive industries. In other words, one would expect removing capital distortions from labor intensive industries to generate small TFP gains, and removing labor distortions from capital intensive industries to generate small TFP gains. In Table 3.2. for a very conservative span of control parameter value, TFP gains with only capital distortions is higher in 8 industries; with labor distortions it's higher in 12 industries. To check whether such gains are due to more misallocation of capital in these industries or the higher weight on capital (large α_s) in capital intensive industries, I plot TFP gains (adjusted by $\overline{\text{TFPR}}_s$) against industry capital intensity (α_s) in Figure 3.7.⁵⁰

⁵⁰ $\overline{\text{TFPR}}_s$ is weighted industry average.

Figure 3.7: TFPR against Capital Intensity



The higher sloped line, representing only capital distortions, indicate that the greater misallocation of capital in capital intensive industries (petroleum, chemicals, tobacco, food and beverages) is due to capital distortions. The TFP gains are relatively more sensitive to capital intensity in capital intensive industries than in labor intensive industries.

3.6 Further Robustness Results

3.6.1 Pooling all firms together

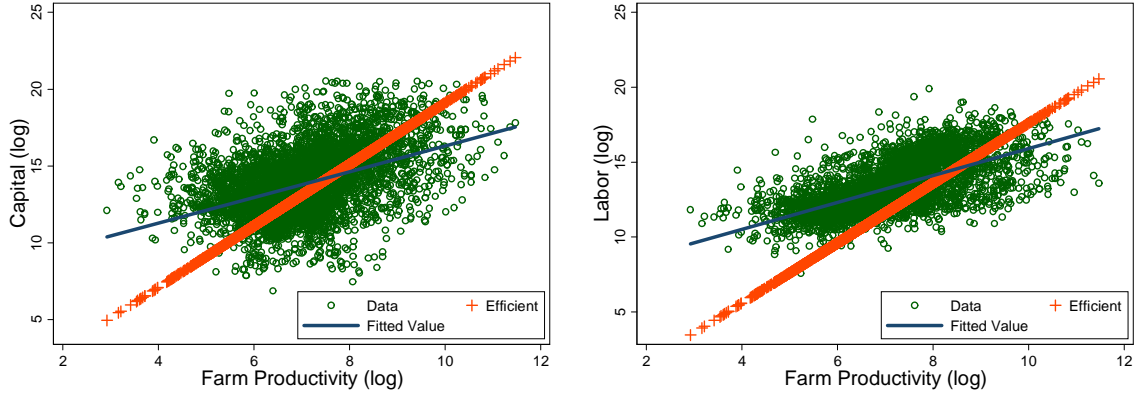
If all 5267 firms were pooled together into a single manufacturing sector and resources were reallocated across the firms, then the estimated TFP^{gain} would be given by

$$\text{TFP}^{\text{gain}} = \frac{\left\{ \sum_{i=1}^M [\text{TFPQ}_i]^{\frac{1}{1-\gamma}} \right\}^{1-\gamma}}{\left\{ \sum_{i=1}^M \left[\text{TFPQ}_i \left(\frac{\overline{\text{TFPR}}}{\text{TFPR}_i} \right)^{\gamma} \right]^{\frac{1}{1-\gamma}} \right\}^{1-\gamma}}.$$

In this experiment I assume $\alpha_s = \alpha = 0.3602$ for all firms, which is a simple average of all shares.⁵¹

In Figure 3.8, I show how allocation is distorted relative to optimal.

Figure 3.8 Distorted Versus Efficient Allocation



The more productive firms do command more capital and labor but the existing allocation is not optimal. If resources could be reallocated proportionately to individual TFPQ_i , then there would be an aggregate TFP gains of 114% for $\gamma = 0.60$, a number that increase to 2.94 fold when $\gamma = 0.60$. TFP gains could be as high as 8.21-fold when $\gamma = 0.80$, a span of control parameter in the range of reasonable estimates in the literature (e.g. Atkeson and Kehoe, 2005).

⁵¹ $\alpha = \frac{\sum \alpha_s}{S}$.

3.6.2 Trimming by 2%

If each industry is trimmed by 2% of relative TFPR, then more firms exposed to high distortions are eliminated. The number of observations drops from 5267 to 5159 in the whole sample. The gap between $\overline{\text{TFPR}}_s$ and TFPR_{si} narrows and the TFP gains lower. For instance, when $\gamma = 0.50$, except for chemicals and tobacco, TFP gains in all industries are less than two fold. There are no changes in TFP in industries with few firms, such as Petroleum, Motor vehicles and Communication Machinery.

3.6.3 Regressing $\ln(\text{TFPR})$ on $\ln(\text{TFPQ})$

By regressing the deviation of $\ln(\text{TFPR})$ from the industry mean on deviation of $\ln(\text{TFPQ})$ from the industry mean, I can check which industry is subject to more distortions. Table 3.4 shows robust regression by industry:

Table 3.4: Regression of $\log(\text{TFPR})$ on $\log(\text{TFPQ})$

Manufacture of	M_s	Elasticity	Standard Error
Food, Beverage	1278	0.7916	0.016
Tobacco	48	0.7792	0.084
Textiles	1510	0.4511	0.015
Apparels	664	0.4771	0.021
Leather	107	0.5895	0.057
Wood	112	0.6608	0.037
Paper	61	0.7164	0.064
Printing	167	0.6103	0.040
Petroleum	11	0.4923	0.230
Chemicals	143	0.6800	0.056
Plastic	127	0.6846	0.050
Non-metal Minerals	496	0.7249	0.026
Metals	60	0.7634	0.053
Fabricated Metals	141	0.7250	0.037
General Machinery	49	0.4946	0.071
Electrical Machinery	19	0.5736	0.113
Communication Machinery	3	0.3394	0.055
Motor Vehicles	6	0.3168	0.259
Transport Equipment	52	0.5956	0.075
Furniture	213	0.5619	0.048
Manufacturing Aggregate	5267	0.5033	0.008

The coefficients are positive and significant for all industries (although there are some industries with very few firms in which cases the estimates are biased). Among the large industries (with many firms), food and beverage and non-metal minerals seem to most distorted; textiles and apparels are relatively more efficient than others. This is not surprising given the exposure of the textile and apparel industry firms to international competition, the additional institutional support at private financial and government levels and the strength of the entrepreneurs.

3.7 Conclusion

Idiosyncratic taxes misallocate factors of production and create heterogeneity in prices across firms within narrowly defined manufacturing industries, curtailing aggregate TFP. There is a growing literature that studies how misallocated resources are responsible for reduced output and aggregate TFP. I have used firm-level microdata to investigate the possible role of misallocation in the manufacturing sector of Bangladesh. I have developed a span-of-control model where production units face capital and labor distortions that firm operators implicitly take into account during resource allocations. Since these distortions are idiosyncratic, average products vary by firm productivity. These misallocations have substantial effects on aggregate output and TFP. The central finding of this chapter is that, in Bangladesh low manufacturing output and productivity is mostly due to misallocation arising from systematic capital distortions across firms within narrowly defined industries.

The strong effect of misallocation (and TFP losses) is perhaps due to capital market imperfections. In Bangladesh, formal lending to manufacturing firms is primarily done by both private and state-owned banks. Although the spread in borrowing rates is small between the rates offered by the these two types of institutions, there is strong evidence of many institutional attributes that make the effective rates very different from each other. For future work, I would like to incorporate into

my model two additional dimensions: 1) the type of credit that firms use (that is, whether it is from state-owned or private banks) 2) divide capital into two parts-the fraction owned by a firm and the fraction that is borrowed. More comprehensive survey has recently been conducted by the BBS on manufacturing firms, and I would like to redo my experiments with a much larger dataset allowing for firms of all sizes.

4 Chapter Four

Dual Economy Misallocation: Micro Evidence from Bangladesh

4.1 Introduction

Can common or sector-specific policies explain the agricultural to non-agricultural productivity gap, that is much higher in poor than in rich countries? The key motivation for this chapter lies in that there are two types of policies and institutions that misallocate resources across production units within sectors. First, there are policies and institutions that affect both agriculture and non-agriculture because by nature they are pervasive in the whole economy. For example, financial frictions, transportation costs, etc. affect not only manufacturing plants also farms. Second, there are policies and institutions that are sector-specific because they affect the allocation of a resource that is predominantly used in one sector. For example, land market institutions and frictions are specific to agriculture as emphasized in the work of Adamopoulos and Restuccia (2014). However, this work is silent about policies that may be impacting non-agriculture through heterogeneous effects across producers. Francisco et al. (2011) emphasize capital market imperfections in manufacturing and services but leave out agriculture. Guner et al. (2008) emphasize size dependent policies in manufacturing and services. This chapter allows for the possibility of misallocation across both agriculture and non-agriculture industries and measures it. Agriculture is a sector that is important for understanding the low income of the poorest nations of the world. But to understand the overall productivity impact of misallocation one cannot leave out the rest of the economy. Given the evidence of misallocation an obvious candidate to consider is capital market institutions, and I ask why common policies across sectors manifest themselves leading to more misallocation in one

sector relative to another. These policies have more detrimental effect on agricultural productivity than on non-agricultural. In this chapter, I explore the quantitative implications of common versus sector-specific misallocation.

I consider a two-sector model of agriculture and non-agriculture featuring: (a) an endogenous distribution of farms in agriculture, and (b) an endogenous distribution of firms in non-agriculture. In each sector the distribution of active production units depends not only on the productivity (ability) of the production unit operator (farmer or entrepreneur respectively) but also on the idiosyncratic distortions that the operator faces in that sector. I capture idiosyncratic distortions in each sector as a producer-sector-specific output "tax" that stands in as a catchall for the policies and institutions that alter the relative prices faced by producers within each sector. The allocation of labor across sectors depends not only on the exogenous sectoral productivity distributions but also on the exogenous sectoral distributions of idiosyncratic distortions. The purpose of the model is to show how policies that introduce common versus sector-specific misallocation affect the structural transformation of the economy, and in particular: the distribution of economy-wide resources (labor and capital) across sectors, the allocation of resources (labor and capital in each sector, as well as land in agriculture) across production units within sectors, aggregate agricultural productivity, aggregate non-agricultural productivity, and aggregate economy-wide productivity. The novelty of the chapter lies in having bivariate distributions of productivity and correlated idiosyncratic distortions in both manufacturing and agriculture and using micro data on firms and farms to discipline the distributions.

The quantitative application of my model is to Bangladesh. I use micro-level data on manufacturing plants and farms and a quantitative framework in order to measure the idiosyncratic distortions within sectors as output "wedges" at the producer-level. I calibrate my two-sector model

to the micro data from Bangladesh with observed distortions. In particular, I calibrate the joint distribution of idiosyncratic distortions and ability within each sector to the observed joint distribution of wedges and production unit level TFP. Then I use my model in order to conduct a set of counterfactual experiments. First, I study the effect of eliminating misallocation in agriculture. Second, I study the effect of eliminating misallocation on non-agriculture. Third, I study the effect of eliminating all misallocation across both sectors. In each case I examine the effect of the within and across sector allocation of resources as well as aggregate productivity. I find that eliminating overall distortions can lead to substantial structural change (in terms of sectoral employment) and increases in sectoral and aggregate productivity.

Given that in this chapter I want to explore the quantitative implications of common versus sector-specific misallocation, I do not put structure on particular policies but leave it for future work. Given my findings an obvious candidate to consider is capital market institutions. An important question that has to be addressed in such an analysis is why common policies or institutions across sectors manifest themselves as leading to more misallocation in one sector relative to another. This is particularly important for developing countries to explain the pattern of these aggregate productivity effects: the agricultural productivity gap (non-agricultural to agricultural labor productivity) is much higher in poor than in rich countries. Why is there a more detrimental effect on agricultural productivity than on non-agricultural for a common institution or policy across sectors?

4.2 Model

In each period the economy produces two consumption goods: an agricultural good (a) and a non-agricultural good (m). The economy is endowed with fixed amounts of total farm land L and

capital K .⁵² The economy is also populated by a stand-in household with a continuum of members of mass one.

Technology in Agriculture: Following Adamopoulos and Restuccia (2014), the production unit in the agricultural sector is a farm. Farm i is a technology that requires the inputs of a farm operator with ability $s_{a,i}$ and land and capital under the farmer's control. The farm-level production technology exhibits decreasing returns to scale and is given by the Cobb-Douglas production function,

$$y_{a,i} = (A_a s_{a,i})^{1-\lambda} (k_{a,i}^\alpha l_i^{1-\alpha})^\lambda \quad (54)$$

where y_a is the output of the farm, l is the amount of land input, and k_a is the amount of capital. A_a is agriculture-specific TFP that affects all farmers in the agricultural sector. Parameter $0 < \lambda < 1$ is the span-of-control parameter that governs returns to scale at the farm level. $0 < \alpha < 1$ is the elasticity of output with respect to capital in the farming technology.

Technology in Non-agriculture: The production unit in the non-agricultural sector is a firm. Firm j is a technology that requires the inputs of an entrepreneur with ability $s_{m,j}$ and labor and capital under the entrepreneur's control. Just as in agriculture, the firm-level production technology exhibits decreasing returns to scale and is given by the Cobb-Douglas production function,

$$y_{m,j} = (A_m s_{m,j})^{1-\gamma} (k_{m,j}^\beta n_j^{1-\beta})^\gamma \quad (55)$$

where y_m is firm output, n is the amount of labour input and k_m is the amount of capital. A_m is

⁵²I abstract from capital accumulation in order to emphasize the direct efficiency effects that result from the reallocation of resources across sectors and across producers within sectors.

non-agriculture-specific TFP that affects all firms in the non-agricultural sector. Decreasing returns to scale at the firm-level are captured by the span of control parameter $0 < \gamma < 1$. $0 < \beta < 1$ is the elasticity of output with respect to capital in the non-agricultural technology.

Preferences and Endowments: The stand-in household, consisting of the continuum of individuals in the economy, has preferences over the agricultural and manufacturing goods by the Stone-Geary utility function,

$$\phi \log(c_a - \bar{a}) + (1 - \phi) \log(c_m) \quad (56)$$

where $\bar{a} > 0$ is a subsistence constraint for agricultural consumption, and $\phi \in (0, 1)$ is a preference weight for the agricultural good. c_a and c_m are per-capita consumption of the agricultural and non-agricultural goods of each household member. These preference capture Engel's law, namely that increases in income are associated with a drop in the consumption of agricultural relative to manufacturing goods.

Each household member is endowed with one unit of productive time that is supplied inelastically to the labor market. The household chooses the shares of its members that will work in the agricultural and manufacturing sectors respectively. All household members are ex-ante identical but become heterogeneous after they are allocated to a given sector. Household members allocated to a given sector are ex-post heterogeneous in two dimensions: (a) their ability (productivity) in the sector they are allocated to, and (b) the idiosyncratic distortions they face in that sector. I abstract from selection in the occupational decision by assuming that the productivity and taxes in each sector are realized after the sectoral allocation decision by the household. In particular, upon entering the agricultural sector a household member draws a vector (s_a, τ_a) , where τ_a is

the farm-specific (idiosyncratic) tax, from a known joint distribution with cdf $\tilde{F}_a(s_a, \tau_a)$ and pdf $\tilde{f}_a(s_a, \tau_a)$. An individual allocated to the manufacturing sector would similarly draw a vector (s_m, τ_m) where τ_m would be the firm-specific tax in manufacturing, from a known joint distribution with cdf $\tilde{F}_m(s_m, \tau_m)$ and pdf $\tilde{f}_m(s_m, \tau_m)$. All individuals allocated to the agricultural sector become farm operators. In other words I abstract from hired labor in agriculture. The reason is that the typical production unit in agriculture is a family farm both in developed and developing countries. Further the type of idiosyncratic distortions I focus on here are not labor-specific. On the other hand, individuals allocated to the manufacturing sector face a choice between becoming entrepreneurs (firm operators) and hired workers. Individuals for which entrepreneurship is less profitable than hired work will choose to become employees at the firms run by those that become entrepreneurs.

I assume that household members face a barrier to the mobility of labor between agriculture and manufacturing. In particular, the return to working in agriculture is a fraction of that in non-agriculture. I introduce this barrier in order to capture that average agricultural labor productivity is lower than non-agricultural labor productivity. This assumption has only implications for the units of measurement of aggregate output. Capital is freely mobile across sectors and across production units within sectors, which implies that all production units face the same rental price of capital r .

Market Structure and Equilibrium I focus on a competitive equilibrium of the model. I assume that the stand-in household, firms in the manufacturing sector, and farms in the agricultural sector behave competitively in factor and output markets. Firm j in manufacturing facing productivity and taxes $(s_{m,j}, \tau_{m,j})$ takes the wage rate w and the rental price of capital r as given and chooses its demand for capital and labor services to maximize profits,

$$\max_{k_{m,j}, n_j} \pi(s_{m,j}, \tau_{m,j}) = \{(1 - \tau_{m,j})(A_m s_{m,j})^{1-\gamma} (k_{m,j}^\beta n_j^{1-\beta})^\gamma - r k_{m,j} - w n_j\}. \quad (57)$$

Farmer i facing productivity and taxes $(s_{a,i}, \tau_{a,i})$ chooses capital and land to maximize profits taking as given the rental prices of land and capital (q, r) and the relative price of the agricultural good p_a ,

$$\max_{k_{a,i}, l_i} \pi(s_{a,i}, \tau_{a,i}) = \{p_a(1 - \tau_{a,i})(A_a s_{a,i})^{1-\gamma} (k_{a,i}^\alpha l_i^{1-\alpha})^\gamma - r k_{a,i} - q l_i\}. \quad (58)$$

The stand-in household maximizes utility in (56) by choosing the consumption allocation across the two goods and the allocation of labor across the two sectors given prices subject to the following budget constraint:

$$p_a c_a + c_n = (1 - N_a)I_m + N_a I_a + qL + rK + T_a + T_m = I \quad (59)$$

where I_a is household income from working in agriculture,

$$I_a = \int_{s_{a,i}} \int_{\tau_{a,i}} \pi(s_{a,i}, \tau_{a,i}) \tilde{f}_a(s_{a,i}, \tau_{a,i}) ds_{a,i} d\tau_{a,i}$$

I_m is total household income from working in manufacturing, which includes both profits of entrepreneurs and wages of workers,

$$\begin{aligned}
I_m = & \int_{s_{m,j}} \int_{\tau_{m,j}} \mathcal{B}(s_{m,j}, \tau_{m,j}) \pi(s_{m,j}, \tau_{m,j}) \tilde{f}_m(s_{m,j}, \tau_{m,j}) ds_{m,j} d\tau_{m,j} \\
& + \int_{s_{m,j}} \int_{\tau_{m,j}} [1 - \mathcal{B}(s_{m,j}, \tau_{m,j})] w \tilde{f}_m(s_{m,j}, \tau_{m,j}) ds_{m,j} d\tau_{m,j}
\end{aligned}$$

where $\mathcal{B}(s_{m,j}, \tau_{m,j})$ is an indicator function that takes the value of 1 if the household member allocated to the manufacturing sector becomes an entrepreneur and 0 otherwise. T_a and T_m are total tax revenues collected from idiosyncratic taxes in agriculture and manufacturing respectively, which are rebated lump-sum to the stand-in household. I denotes aggregate (household) income for this economy.

A competitive equilibrium is a set of allocations for:

- (a) the household $\{c_a, c_n, N_a\}$
- (b) entrepreneurs in the manufacturing sector $\{[y_m(s_m, \tau_m), k_m(s_m, \tau_m), n_m(s_m, \tau_m)]_{S_m \times T_m}\}$,
and
- (c) farmers in the agricultural sector $\{[y_a(s_a, \tau_a), k_a(s_a, \tau_a), l_a(s_a, \tau_a)]_{S_a \times T_a}\}$, and a set of prices $\{p_a, q, r, w\}$ such that:
 - (i) given prices, the allocations of the household solve the household's problem, i.e., maximize utility in (56) subject to the budget constraint in (59)
 - (ii) given prices, the allocations of firms in manufacturing and farms in agriculture solve their problems in (57) and (58) respectively;
 - (iii) all markets clear:

- for labor,

$$N_a + (1 - N_a) \int_{s_{m,j}} \int_{\tau_{m,j}} \mathcal{B}(s_{m,j}, \tau_{m,j}) \pi(s_{m,j}, \tau_{m,j}) \tilde{f}_m(s_{m,j}, \tau_{m,j}) ds_{m,j} d\tau_{m,j}$$

$$+ (1 - N_a) \int_{s_{m,j}} \int_{\tau_{m,j}} \mathcal{B}(s_{m,j}, \tau_{m,j}) n(s_{m,j}, \tau_{m,j}) \tilde{f}_m(s_{m,j}, \tau_{m,j}) ds_{m,j} d\tau_{m,j} = 1$$

- for capital,

$$K_a + K_m = K$$

where

$$K_a = N_a \int_{s_{a,i}} \int_{\tau_{a,i}} k_a(s_{a,i}, \tau_{a,i}) \tilde{f}_a(s_{a,i}, \tau_{a,i}) ds_{a,i} d\tau_{a,i}$$

$$K_m = (1 - N_a) \int_{s_{m,j}} \int_{\tau_{m,j}} \mathcal{B}(s_{m,j}, \tau_{m,j}) k_m(s_{m,j}, \tau_{m,j}) \tilde{f}_m(s_{m,j}, \tau_{m,j}) ds_{m,j} d\tau_{m,j}$$

- for land,

$$L = N_a \int_{s_{a,i}} \int_{\tau_{a,i}} l_a(s_{a,i}, \tau_{a,i}) \tilde{f}_a(s_{a,i}, \tau_{a,i}) ds_{a,i} d\tau_{a,i}$$

- for agricultural goods,

$$c_a = N_a \int_{s_{a,i}} \int_{\tau_{a,i}} y_a(s_{a,i}, \tau_{a,i}) \tilde{f}_a(s_{a,i}, \tau_{a,i}) ds_{a,i} d\tau_{a,i} = Y_a$$

- for manufacturing goods,

$$c_m = (1 - N_a) \int_{s_{m,j}} \int_{\tau_{m,j}} \mathcal{B}(s_{m,j}, \tau_{m,j}) y_m(s_{m,j}, \tau_{m,j}) \tilde{f}_m(s_{m,j}, \tau_{m,j}) ds_{m,j} d\tau_{m,j} = Y_m$$

(iv) total transfers are equal to total tax revenues collected,

$$T_a = N_a \int_{s_{a,i}} \int_{\tau_{a,i}} \tau_{a,i} \cdot p_a \cdot y_a(s_{a,i}, \tau_{a,i}) \tilde{f}_a(s_{a,i}, \tau_{a,i}) ds_{a,i} d\tau_{a,i}$$

$$T_m = (1 - N_a) \int_{s_{m,j}} \int_{\tau_{m,j}} \mathcal{B}(s_{m,j}, \tau_{m,j}) \cdot \tau_{m,j} \cdot y_m(s_{m,j}, \tau_{m,j}) \tilde{f}_m(s_{m,j}, \tau_{m,j}) ds_{m,j} d\tau_{m,j}$$

(d) and indicator function $\mathcal{B}(s_{m,j}, \tau_{m,j})$ such that

$$(i) \mathcal{B}(s_{m,j}, \tau_{m,j}) = 1 \text{ if } \pi(s_{m,j}, \tau_{m,j}) > w.$$

4.3 Characterization

Here I characterize the equilibrium defined in the previous section. I denote the wedge faced by farmer i in agriculture by $\varphi_{a,i} \equiv (1 - \tau_{a,i})$. The profit maximization problem of a farmer i that faces ability $s_{a,i}$ and taxes $\tau_{a,i}$ implies demand for land and capital,

$$l(s_{a,i}, \varphi_{a,i}) = A_a (\lambda p_a)^{\frac{1}{1-\gamma}} \left(\frac{1-\alpha}{q} \right)^{\frac{1-\alpha\lambda}{1-\lambda}} \left(\frac{\alpha}{r} \right)^{\frac{\alpha\lambda}{1-\lambda}} \varphi_{a,i}^{\frac{1}{1-\gamma}} s_{a,i} \quad (60)$$

$$k_a(s_{a,i}, \varphi_{a,i}) = A_a (\lambda p_a)^{\frac{1}{1-\gamma}} \left(\frac{1-\alpha}{q} \right)^{\frac{\lambda(1-\alpha)}{1-\lambda}} \left(\frac{\alpha}{r} \right)^{\frac{1-\lambda(1-\alpha)}{1-\lambda}} \varphi_{a,i}^{\frac{1}{1-\gamma}} s_{a,i} \quad (61)$$

The amount of agricultural output supplied and profits made by farmer i are,

$$y_a(s_{a,i}, \varphi_{a,i}) = A_a (\lambda p_a)^{\frac{\lambda}{1-\gamma}} \left(\frac{1-\alpha}{q} \right)^{\frac{\lambda(1-\alpha)}{1-\lambda}} \left(\frac{\alpha}{r} \right)^{\frac{\alpha\lambda}{1-\lambda}} \varphi_{a,i}^{\frac{\lambda}{1-\lambda}} s_{a,i}$$

$$\pi_a(s_{a,i}, \varphi_{a,i}) = A_a (1-\lambda) \lambda^{\frac{\lambda}{1-\lambda}} p_a^{\frac{1}{1-\lambda}} \left(\frac{1-\alpha}{q} \right)^{\frac{\lambda(1-\alpha)}{1-\lambda}} \left(\frac{\alpha}{r} \right)^{\frac{\alpha\lambda}{1-\lambda}} \varphi_{a,i}^{\frac{1}{1-\lambda}} s_{a,i}$$

Notice that demand for land, demand for capital, output supply and profits are all increasing (linear) functions of farm productivity and decreasing functions of taxes. Note that a more productive farmer may not necessarily command more resources and produce more if faced with steep taxes. The allocation of resources within agriculture depends on the configuration farm productivity and farm-specific distortions. Similarly I denote the wedge faced by entrepreneur j in manufacturing by $\varphi_{m,j} \equiv (1 - \tau_{m,j})$. The profit maximization problem of an entrepreneur j with ability $s_{m,j}$ and wedge $\varphi_{m,j}$ imply the following demand for labor, demand for capital, output supply and profit

$$n(s_{m,j}, \varphi_{m,j}) = A_m \gamma^{\frac{1}{1-\gamma}} \left(\frac{1-\beta}{w} \right)^{\frac{1-\beta\gamma}{1-\gamma}} \left(\frac{\beta}{r} \right)^{\frac{\beta\gamma}{1-\gamma}} \varphi_{m,j}^{\frac{1}{1-\gamma}} s_{m,j} \quad (62)$$

$$k_m(s_{m,j}, \varphi_{m,j}) = A_m \gamma^{\frac{1}{1-\gamma}} \left(\frac{1-\beta}{w} \right)^{\frac{\gamma(1-\beta)}{1-\gamma}} \left(\frac{\beta}{r} \right)^{\frac{1-\gamma(1-\beta)}{1-\gamma}} \varphi_{m,j}^{\frac{1}{1-\gamma}} s_{m,j} \quad (63)$$

$$y_m(s_{m,j}, \varphi_{m,j}) = A_m \gamma^{\frac{\gamma}{1-\gamma}} \left(\frac{1-\beta}{w} \right)^{\frac{\gamma(1-\beta)}{1-\gamma}} \left(\frac{\beta}{r} \right)^{\frac{\beta\gamma}{1-\gamma}} \varphi_{m,j}^{\frac{\gamma}{1-\gamma}} s_{m,j} \quad (64)$$

$$\pi_m(s_{m,j}, \varphi_{m,j}) = A_m (1-\gamma) \gamma^{\frac{\gamma}{1-\gamma}} \left(\frac{1-\beta}{w} \right)^{\frac{\gamma(1-\beta)}{1-\gamma}} \left(\frac{\beta}{r} \right)^{\frac{\beta\gamma}{1-\gamma}} \varphi_{m,j}^{\frac{1}{1-\gamma}} s_{m,j} \quad (65)$$

Again the more productive entrepreneurs and those facing lower taxes will hire more labor and capital, produce more output, and make more profit. The household's first order conditions imply the following choices with respect to consumption of agricultural and manufacturing goods,

$$\begin{aligned}
c_a &= \bar{a} + \frac{\phi I}{p_a}(I - \bar{a}p_a) \\
c_m &= \frac{(1 - \phi)}{p_a}(I - \bar{a}p_a)
\end{aligned}$$

These imply that when income is low the household devotes a disproportionate amount to the consumption of agricultural goods. The household's allocation of labor across sectors is governed by the following no-arbitrage condition,

$$\begin{aligned}
\int_{s_{a,i}} \int_{\varphi_{a,i}} \pi(s_{a,i}, \varphi_{a,i}) d\tilde{F}_a(s_{a,i}, \varphi_{a,i}) &= \int_{s_{m,j}} \int_{\varphi_{m,j}} [1 - \mathcal{B}(s_{m,j}, \varphi_{m,j})] w d\tilde{F}_m(s_{m,j}, \varphi_{m,j}) + \\
&\quad \int_{s_{m,j}} \int_{\varphi_{m,j}} \mathcal{B}(s_{m,j}, \varphi_{m,j}) \pi(s_{m,j}, \varphi_{m,j}) d\tilde{F}_m(s_{m,j}, \varphi_{m,j})
\end{aligned}$$

which says that expected (average) profit from becoming a farmer in agriculture should be equal to expected (average) income from becoming an entrepreneur and making profits or a hired worker and earning wages in manufacturing.

4.4 Calibration

I calibrate a model economy with distortions to the establishment-level and farm-level data of Bangladesh. My strategy is to calibrate some parameters based on a priori information, and determine values for the rest of the parameters to match targets in the data for Bangladesh so that the solution of the baseline economy constitutes an equilibrium.

The distributions of active production units in each sector, manufacturing and agriculture, is determined by the joint distribution of productivity and distortions within that sector. I proceed

as follows. First, I assume that the within-sector bivariate distributions between productivity and distortions, $\tilde{F}_m(s_{m,j}, \varphi_{m,j})$ in manufacturing, and $\tilde{F}_a(s_{a,i}, \varphi_{a,i})$ in agriculture are each log-normal. In particular, productivity and distortions in manufacturing are jointly log-normally distributed,

$$(s_m, \varphi_m) \sim LN(\mathcal{M}_m, \Sigma_m)$$

where \mathcal{M}_m and Σ_m are the vector of means and the variance-covariance matrix of the log-normal distribution

$$\mathcal{M}_m = \begin{pmatrix} \mu_m \\ \mu_{\varphi_m} \end{pmatrix}, \Sigma_m = \begin{pmatrix} \sigma_m^2 & \sigma_{m\varphi_m} \\ \sigma_{m\varphi_m} & \sigma_{\varphi_m}^2 \end{pmatrix}$$

Similarly the joint distribution of productivity and distortions in agriculture is given by,

$$(s_a, \varphi_a) \sim LN(\mathcal{M}_a, \Sigma_a)$$

with the corresponding \mathcal{M}_a and Σ_a given by,

$$\mathcal{M}_a = \begin{pmatrix} \mu_a \\ \mu_{\varphi_a} \end{pmatrix}, \Sigma_a = \begin{pmatrix} \sigma_a^2 & \sigma_{a\varphi_a} \\ \sigma_{a\varphi_a} & \sigma_{\varphi_a}^2 \end{pmatrix}$$

Note that I allow for the possibility that distortions and productivity are correlated, with the direction and extent being determined from the micro data. Also note that consistent with the model, the two sectoral bivariate log-normal distributions are independent from each other.

Second, I construct establishment-level and farm-level TFP (equivalent to TFPQ in previous chapters) for each production unit in our data as a residual from the unit-level production functions,

$$TFP_{m,j} = (A_m s_{m,j})^{1-\gamma} = \frac{y_{m,j}}{(k_{m,j}^\beta n_j^{1-\beta})^\gamma},$$

$$TFP_{a,i} = (A_a s_{a,i})^{1-\lambda} = \frac{y_{a,i}}{(k_{a,i}^\alpha l_i^{1-\alpha})^\lambda}.$$

Third, I construct a summary measure of establishment-specific wedges for each establishment and a summary measure of farm-specific wedges for each farm in my data set. The wedges for manufacturing establishments summarize for each establishment distortions to capital and labor, backed out as deviations of actual from optimal allocations. In particular, the summary establishment-specific wedge on entrepreneur j is

$$\varphi_{m,j} = \frac{1}{\left(1 + \tau_{m,j}^k\right)^\beta \left(1 + \tau_{m,j}^n\right)^{1-\beta}}$$

where $(\tau_{m,j}^k, \tau_{m,j}^n)$ are the establishment-specific wedges on capital and labor constructed in Chapter 3.⁵³ Similarly, I construct summary measures for farm-specific wedges for each farm in my data set from the farm-level wedges on capital and land constructed in Chapter 2

$$\varphi_{a,i} = \frac{1}{\left(1 + \tau_{a,i}^k\right)^\alpha \left(1 + \tau_{a,i}^l\right)^{1-\alpha}}.$$

Then I calculate for each sector, the means and standard deviations of log productivity and log wedges, as well as the covariance of log productivity and log wedges. The empirical moments are in Table 4.1.

⁵³Derivation of farm and firm-level wedges on capital and land is in the Appendix for Chapter 4.

I calibrate the elasticity parameters in the production functions to U.S. factor shares. The reason for this approach is that the elasticity parameters represent technological parameters and distortions in Bangladesh would confound measured factor shares. By pinning down these parameters from U.S. data I am implicitly assuming that the U.S. is an economy with relatively few idiosyncratic distortions. Following Adamopoulos and Restuccia (2014) I set $\lambda = 0.54$ to match the share of total capital (land and physical capital) for the U.S. economy. I then choose $\alpha = 0.63$ to match a share of land in total capital of 20% for the U.S. economy (Valentinyi and Herrendorf, 2008). I set the establishment-level span of control parameter in manufacturing $\gamma = 0.85$ to match the extent of decreasing returns to scale in U.S. manufacturing of 0.85 (see for example Atkeson and Kehoe, 2005). Finally I choose $\beta = 0.39$ to match a capital income share of $1/3$ as is standard for the U.S. economy.

In the benchmark economy I normalize the relative price of agriculture to 1. I also normalize the manufacturing-specific TFP parameter that affects all establishments in manufacturing to 1. I choose L to match an average farm size for Bangladesh of 0.35 hectares (2005 World Census of Agriculture, Food and Agricultural Organization).

Table 4.1 Empirical Moments from Micro Data

Manufacturing: Establishment-level data	
Moment	Data
Mean of log-productivity	-1.75
Mean of log-distortions	-0.21
Standard deviation of log-productivity	0.88
Standard deviation of log-distortions	0.90
Covariance of log-productivity and log distortions	-0.76
Agriculture: Farm-level data	
Moment	Data
Mean of log-productivity	-0.86
Mean of log-distortions	-0.32
Standard deviation of log-productivity	0.85
Standard deviation of log-distortions	1.05
Covariance of log-productivity and log-distortions	-0.77

Note: Log-productivity in manufacturing is $\log(TFP_{m,j})$; Log-distortions in manufacturing are $\log(\varphi_{m,j})$; Log-productivity in agriculture is $\log(TFP_{a,i})$; Log-distortions in agriculture are $\log(\varphi_{a,i})$.

I set $\phi = 0.010$ to match a long-run share of employment in agriculture of 1%. I then solve the model for the remaining three parameters: the subsistence constraint for food \bar{a} , the barrier to the mobility of labor across sectors η , the agriculture-specific TFP parameter (common to all farms in agriculture) A_a to match three targets for the economy of Bangladesh: (a) a share of employment in agriculture of 78% (2010 World Development Indicators, World Bank)⁵⁴; (b) a ratio of average manufacturing to average agricultural labor productivity of 2.8 (average from aggregate 2010 data from BBS and micro data); and (c) a capital-output ratio in agriculture of 2.3 (average from 2005 FAOSTAT and the farm-level data). The parameter η is the barrier to the mobility of workers from agriculture to non-agriculture. It matters only for units of measurement of aggregate output so that aggregate labor productivity in non-agriculture relative to agriculture labor productivity is what it is in the data $\left(\frac{Y_m/N_m}{Y_a/N_a}\right)$. Without this constraint, the agricultural sector might end up being

⁵⁴The share of agriculture in total employment is 48% and that of industry is 17.7%. Given that in our model there is no services sector, agriculture accounts for 73% of employment in the goods producing sectors.

more productive than the non-agricultural sector. With re-allocation of labor from agriculture to non-agriculture aggregate output would fall.

Table 4.2 Parameterization

Parameter	Value	Target
A_m	1	Normalization
A_a	0.25	Capital-output in agriculture
β	0.39	Agricultural capital income share
γ	0.85	Decreasing returns to scale in manufacturing
α	0.63	Agricultural land income share
λ	0.54	Agricultural capital income share
\bar{a}	0.20	Current employment share in agriculture
ϕ	0.010	Long-run employment share in agriculture
L	0.25	Average farm size
η	0.15	Manufacturing-Agriculture productivity ratio

4.5 Quantitative Experiments

Next I consider a set of counterfactual experiments in order to understand the importance of idiosyncratic distortions in manufacturing and agriculture on the sectoral allocation of labor, sectoral labor productivity, and aggregate productivity. Starting from the baseline calibrated economy to Bangladesh, I conduct three counterfactual experiments by eliminating in turn, idiosyncratic distortions in agriculture, idiosyncratic distortions in manufacturing, and all idiosyncratic distortions (across both sectors). I report the results of these experiments in Table 4.3. Agricultural and manufacturing labor productivity are both in real terms and output is also expressed in real terms by using a common relative price (equal to the value in the baseline economy) to aggregate output across sectors. In order to emphasize the changes relative to the benchmark economy I normalize the value of average establishment size in manufacturing, average farm size in agriculture, agricultural, manufacturing and aggregate labor productivity to 1 in the benchmark economy (column 1). The second, third and fourth columns indicate factor differences relative to the benchmark

economy and the share of employment in agriculture from eliminating idiosyncratic distortions in agriculture, manufacturing, and both. On the one hand, sector-specific distortions, such as land market institutions in agriculture or labor policies for industrial workers, would show up only in the idiosyncratic distortions of each sector. On the other hand, common distortions across sectors (that manifest themselves in idiosyncratic fashion with respect to production units) such as financial frictions would show up in both sectors. My experiments can thus be viewed through these lens, as shedding light on the relative importance of common versus sector-specific distortions.

I find that eliminating distortions in agriculture (column 2) would raise agricultural productivity by 22% and reduce the share of employment in agriculture from 73% to 61%. However, productivity in manufacturing would in fact fall slightly, implying that aggregate output per worker would increase by 18%. Eliminating distortions in manufacturing (column 3) would increase manufacturing productivity by 22% and aggregate productivity by 13% but have virtually no effect on agricultural productivity or the share of employment in agriculture. Not surprisingly, when all distortions are eliminated in both sectors (column 4) would raise aggregate productivity by 39%, which is due to an increase in agricultural and manufacturing productivity, as well as a drop in the share of labor in agriculture, indicating a shift to the more productive sector.

Table 4.3 Effects of Eliminating Distortions

	Benchmark Economy	Eliminate Agri. Dist.	Eliminate Manu Dist.	Eliminate All Dist.
Employment in Agriculture	73.0	61.5	72.6	61.3
Average Establishment Size	1.0	1.0	0.98	0.90
Average Farm Size	1.0	1.19	1.0	1.19
Labor Prod. in Manu. (Y_m/N_m)	1.0	0.94	1.22	1.23
Labor Prod. in Agriculture (Y_a/N_a)	1.0	1.21	1.0	1.21
Aggregate Labor Productivity (Y/N)	1.0	1.18	1.13	1.39

Note: Average establishment size, average farm size, real labor productivity in agriculture, real labor productivity in manufacturing, and real aggregate labor productivity are reported as the ratio between the counterfactual economy and the benchmark economy.

4.6 Conclusion

I calibrated a two-sector model of agriculture and manufacturing to an economy with distortions, and conducted a set of counterfactual experiments where I shut down distortions by sector as well as jointly. I took into account the general equilibrium effects generated by the elimination of the distortions. I found that eliminating distortions could lead to substantial structural change, reducing the share of agriculture (in terms of employment) from 73% to 61% and increase output per worker by about 40%.

Given that the goal of this chapter was to explore the quantitative implications of common versus sector-specific misallocation I did not put structure on particular policies, but leave it for future work. Given my findings an obvious candidate to consider is capital market institutions. An important question that has to be addressed in such an analysis is why common policies or institutions across sectors manifest themselves as leading to more misallocation in one sector relative to another? This is particularly important for developing countries to explain the pattern of these aggregate productivity effects: the agricultural productivity gap (non-agricultural to agricultural labor productivity) is much higher in poor than in rich countries. Why is there a more detrimental effect on agricultural productivity than on non-agricultural for a common institution or policy across sectors?

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A Appendix for Chapter 2

Technology of a farm in agriculture:

$$y_i = s_i^{1-\gamma} \left[\left(k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta} \right)^{1-\mu} x_i^\mu \right]^\gamma$$

Profit of a farm:

$$\begin{aligned} \pi_i &= py_i - (1 + \tau_i^k)rk_i - (1 + \tau_i^l)ql_i - (1 + \tau_i^n)wn_i - (1 + \tau_i^x)zx_i \\ &= ps_i^{1-\gamma} \left[\left(k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta} \right)^{1-\mu} x_i^\mu \right]^\gamma - (1 + \tau_i^k)rk_i - (1 + \tau_i^l)ql_i - (1 + \tau_i^n)wn_i - (1 + \tau_i^x)zx_i \end{aligned}$$

First order conditions and marginal revenue products:

$$\begin{aligned} \frac{\partial \pi_i}{\partial k_i} &= \underbrace{\frac{ps_i^{1-\gamma} \left[\left(k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta} \right)^{1-\mu} x_i^\mu \right]^\gamma \gamma(1-\mu)\alpha}{k_i}}_{\text{MRPK}_i} = (1 + \tau_i^k)r \\ \text{MRPK}_i &\equiv \gamma(1-\mu)\alpha \frac{py_i}{k_i} = (1 + \tau_i^k)r \end{aligned}$$

$$\begin{aligned} \frac{\partial \pi_i}{\partial l_i} &= \underbrace{\frac{ps_i^{1-\gamma} \left[\left(k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta} \right)^{1-\mu} x_i^\mu \right]^\gamma \gamma(1-\mu)\beta}{l_i}}_{\text{MRPL}_i} = (1 + \tau_i^l)q \\ \text{MRPL}_i &\equiv \gamma(1-\mu)\beta \frac{py_i}{l_i} = (1 + \tau_i^l)q \end{aligned}$$

$$\frac{\partial \pi_i}{\partial n_i} = \frac{ps_i^{1-\gamma} \left[\left(k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta} \right)^{1-\mu} x_i^\mu \right]^\gamma \gamma(1-\mu)(1-\alpha-\beta)}{\underbrace{n_i}_{\text{MRPN}_i}} = (1 + \tau_i^n)w$$

$$\text{MRPN}_i \equiv \gamma(1-\mu)(1-\alpha-\beta) \frac{py_i}{n_i} = (1 + \tau_i^n)w$$

$$\frac{\partial \pi_i}{\partial x_i} = \frac{ps_i^{1-\gamma} \left[\left(k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta} \right)^{1-\mu} x_i^\mu \right]^\gamma \gamma\mu}{\underbrace{x_i}_{\text{MRPX}_i}} = (1 + \tau_i^x)z$$

$$\text{MRPX}_i \equiv \gamma\mu \frac{py_i}{x_i} = (1 + \tau_i^x)z$$

Using the first order conditions the factor ratios are:

$$\text{capital land ratio} \quad \frac{k_i}{l_i} = \frac{\alpha}{\beta} \frac{(1+\tau_i^l)}{(1+\tau_i^k)} \frac{q}{r}$$

$$\text{capital-labor ratio} \quad \frac{k_i}{n_i} = \frac{\alpha}{1-\alpha-\beta} \frac{(1+\tau_i^n)}{(1+\tau_i^k)} \frac{w}{r}$$

$$\text{land-labor ratio} \quad \frac{l_i}{n_i} = \frac{\beta}{1-\alpha-\beta} \frac{(1+\tau_i^n)}{(1+\tau_i^l)} \frac{w}{q}$$

$$\text{capital-input ratio} \quad \frac{k_i}{x_i} = \frac{\alpha(1-\mu)}{\mu} \frac{(1+\tau_i^x)}{(1+\tau_i^k)} \frac{z}{r}$$

$$\text{labor-input ratio} \quad \frac{n_i}{x_i} = (1-\alpha-\beta) \frac{(1-\mu)}{\mu} \frac{(1+\tau_i^x)}{(1+\tau_i^n)} \frac{z}{w}$$

Derivation of equilibrium factor demands:

From $MRPN_i$

$$ps_i^{1-\gamma} \left[\left(k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta} \right)^{1-\mu} x_i^\mu \right]^\gamma \gamma(1-\mu)(1-\alpha-\beta) = (1+\tau_i^n)wn_i$$

$$ps_i^{1-\gamma} \left\{ \left[\left(\frac{k}{n} \right)^\alpha \left(\frac{l}{n} \right)^\beta n_i \right]^{1-\mu} x_i^\mu \right\}^\gamma \gamma(1-\mu)(1-\alpha-\beta) = (1+\tau_i^n)wn_i$$

$$ps_i^{1-\gamma} \left\{ \left[\left(\frac{\alpha}{1-\alpha-\beta} \frac{(1+\tau_i^n)w}{(1+\tau_i^k)r} \right)^\alpha \left(\frac{\beta}{1-\alpha-\beta} \frac{(1+\tau_i^n)w}{(1+\tau_i^l)q} \right)^\beta n_i \right]^{1-\mu} x_i^\mu \right\}^\gamma \gamma(1-\mu)(1-\alpha-\beta) = (1+\tau_i^n)wn_i$$

$$ps_i^{1-\gamma} \left\{ \left[\left(\frac{\alpha}{1-\alpha-\beta} \frac{(1+\tau_i^n)w}{(1+\tau_i^k)r} \right)^\alpha \left(\frac{\beta}{1-\alpha-\beta} \frac{(1+\tau_i^n)w}{(1+\tau_i^l)q} \right)^\beta \right]^{1-\mu} \right\}^\gamma \gamma(1-\mu)(1-\alpha-\beta)x_i^{\mu\gamma} = (1+\tau_i^n)wn_i^{1-\gamma+\mu\gamma}$$

$$\frac{n_i}{x_i} = \frac{(1-\mu)(1-\alpha-\beta)}{\mu} \frac{(1+\tau_i^x)z}{(1+\tau_i^n)w} \implies \frac{n_i}{(1-\alpha-\beta)} \frac{\mu}{(1-\mu)} \frac{(1+\tau_i^n)w}{(1+\tau_i^x)z} = x_i$$

$$ps_i^{1-\gamma} \left\{ \left[\left(\frac{\alpha}{1-\alpha-\beta} \frac{(1+\tau_i^n)w}{(1+\tau_i^k)r} \right)^\alpha \left(\frac{\beta}{1-\alpha-\beta} \frac{(1+\tau_i^n)w}{(1+\tau_i^l)q} \right)^\beta \right]^{1-\mu} \right\}^\gamma.$$

$$\gamma(1-\mu)(1-\alpha-\beta) \left[\frac{n_i}{(1-\alpha-\beta)} \frac{\mu}{(1-\mu)} \frac{(1+\tau_i^n)w}{(1+\tau_i^x)z} \right]^{\mu\gamma} = (1+\tau_i^n)wn_i^{1-\gamma+\mu\gamma}$$

$$p\gamma s_i^{1-\gamma} \left[\frac{\alpha^\alpha \beta^\beta}{(1-\alpha-\beta)^{\alpha+\beta}} \frac{w^{\alpha+\beta}}{r^\alpha q^\beta} \right]^{(1-\mu)\gamma} \frac{(1+\tau_i^n)^{\alpha(1-\mu)\gamma+\beta(1-\mu)\gamma+\mu\gamma-1}}{(1+\tau_i^k)^{\alpha(1-\mu)\gamma}(1+\tau_i^l)^{\beta(1-\mu)\gamma}(1+\tau_i^x)^{\mu\gamma}}.$$

$$\mu^{\mu\gamma}(1-\mu)^{1-\mu\gamma}(1-\alpha-\beta)^{1-\mu\gamma} \left(\frac{w}{z} \right)^{\mu\gamma} = wn_i^{1-\gamma}$$

$$p\gamma \mu^{\mu\gamma}(1-\mu)^{1-\mu\gamma} s_i^{1-\gamma} \frac{[\alpha^\alpha \beta^\beta]^{(1-\mu)\gamma}}{(1-\alpha-\beta)^{(\alpha+\beta)(1-\mu)\gamma+\mu\gamma-1}} w^{\alpha(1-\mu)\gamma+\beta(1-\mu)\gamma+\mu\gamma-1} \left[\frac{1}{r^\alpha q^\beta} \right]^{(1-\mu)\gamma}.$$

$$\left(\frac{1}{z} \right)^{\mu\gamma} \frac{(1+\tau_i^n)^{\alpha(1-\mu)\gamma+\beta(1-\mu)\gamma+\mu\gamma-1}}{(1+\tau_i^k)^{\alpha(1-\mu)\gamma}(1+\tau_i^l)^{\beta(1-\mu)\gamma}(1+\tau_i^x)^{\mu\gamma}} = n_i^{1-\gamma}$$

$$n_i = s_i \left\{ p\gamma\mu^{\mu\gamma}(1-\mu)^{1-\mu\gamma} \left[\frac{w}{(1-\alpha-\beta)} \right]^{(\alpha+\beta)(1-\mu)\gamma+\mu\gamma-1} \left[\left(\frac{\alpha}{r} \right)^\alpha \left(\frac{\beta}{q} \right)^\beta \right]^{(1-\mu)\gamma} \right. \\ \left. \left(\frac{1}{z} \right)^{\mu\gamma} \frac{(1+\tau_i^n)^{\alpha(1-\mu)\gamma+\beta(1-\mu)\gamma+\mu\gamma-1}}{(1+\tau_i^k)^{\alpha(1-\mu)\gamma}(1+\tau_i^l)^{\beta(1-\mu)\gamma}(1+\tau_i^x)^{\mu\gamma}} \right\}^{\frac{1}{1-\gamma}}$$

Substituting n_i into the capital-labor ratio to solve for k_i : $k_i = \frac{\alpha}{1-\alpha-\beta} \frac{(1+\tau_i^n)}{(1+\tau_i^k)} \frac{w}{r} n_i$

$$k_i = \left\{ \frac{\alpha}{1-\alpha-\beta} \frac{(1+\tau_i^n)}{(1+\tau_i^k)} \frac{w}{r} \right\} s \left\{ p\gamma\mu^{\mu\gamma}(1-\mu)^{1-\mu\gamma} \left[\frac{w}{(1-\alpha-\beta)} \right]^{(\alpha+\beta)(1-\mu)\gamma+\mu\gamma-1} \right. \\ \left. \left[\left(\frac{\alpha}{r} \right)^\alpha \left(\frac{\beta}{q} \right)^\beta \right]^{(1-\mu)\gamma} \left(\frac{1}{z} \right)^{\mu\gamma} \frac{(1+\tau_i^n)^{\alpha(1-\mu)\gamma+\beta(1-\mu)\gamma+\mu\gamma-1}}{(1+\tau_i^k)^{\alpha(1-\mu)\gamma}(1+\tau_i^l)^{\beta(1-\mu)\gamma}(1+\tau_i^x)^{\mu\gamma}} \right\}^{\frac{1}{1-\gamma}} \\ = s_i \left\{ p\gamma\mu^{\mu\gamma}(1-\mu)^{1-\mu\gamma} \left[\frac{w}{(1-\alpha-\beta)} \right]^{(1-\mu)\gamma(\alpha+\beta-1)} \left(\frac{\beta}{q} \right)^{\beta(1-\mu)\gamma} \left(\frac{\alpha}{r} \right)^{\alpha(1-\mu)\gamma+1-\gamma} \right. \\ \left. \left(\frac{1}{z} \right)^{\mu\gamma} \frac{(1+\tau_i^n)^{(1-\mu)\gamma(\alpha+\beta-1)}}{(1+\tau_i^k)^{\alpha(1-\mu)\gamma+1-\gamma}(1+\tau_i^l)^{\beta(1-\mu)\gamma}(1+\tau_i^x)^{\mu\gamma}} \right\}^{\frac{1}{1-\gamma}}$$

Substituting n_i into the capital-labor ratio to solve for l_i : $l_i = \frac{\beta}{1-\alpha-\beta} \frac{(1+\tau_i^n)}{(1+\tau_i^l)} \frac{w}{q} n_i$

$$l_i = \left\{ \frac{\beta}{1-\alpha-\beta} \frac{(1+\tau_i^n)}{(1+\tau_i^l)} \frac{w}{q} \right\} s \left\{ p\gamma\mu^{\mu\gamma}(1-\mu)^{1-\mu\gamma} \left[\frac{w}{(1-\alpha-\beta)} \right]^{(\alpha+\beta)(1-\mu)\gamma+\mu\gamma-1} \right. \\ \left. \left[\left(\frac{\alpha}{r} \right)^\alpha \left(\frac{\beta}{q} \right)^\beta \right]^{(1-\mu)\gamma} \left(\frac{1}{z} \right)^{\mu\gamma} \frac{(1+\tau_i^n)^{\alpha(1-\mu)\gamma+\beta(1-\mu)\gamma+\mu\gamma-1}}{(1+\tau_i^k)^{\alpha(1-\mu)\gamma}(1+\tau_i^l)^{\beta(1-\mu)\gamma}(1+\tau_i^x)^{\mu\gamma}} \right\}^{\frac{1}{1-\gamma}} \\ = s_i \left\{ p\gamma\mu^{\mu\gamma}(1-\mu)^{1-\mu\gamma} \left[\frac{w}{(1-\alpha-\beta)} \right]^{(1-\mu)\gamma(\alpha+\beta-1)} \left(\frac{\alpha}{r} \right)^{\alpha(1-\mu)\gamma} \left(\frac{\beta}{q} \right)^{\beta(1-\mu)\gamma+1-\gamma} \right. \\ \left. \left(\frac{1}{z} \right)^{\mu\gamma} \frac{(1+\tau_i^n)^{(1-\mu)\gamma(\alpha+\beta-1)}}{(1+\tau_i^k)^{\alpha(1-\mu)\gamma}(1+\tau_i^l)^{\beta(1-\mu)\gamma+1-\gamma}(1+\tau_i^x)^{\mu\gamma}} \right\}^{\frac{1}{1-\gamma}}$$

Substituting n_i into the capital-labor ratio to solve for x_i : $x_i = \frac{1}{(1-\alpha-\beta)} \frac{\mu}{(1-\mu)} \frac{(1+\tau_i^n)}{(1+\tau_i^x)} \frac{w}{z} n_i$

$$\begin{aligned}
x_i &= \left\{ \frac{1}{(1-\alpha-\beta)} \frac{\mu}{(1-\mu)} \frac{(1+\tau_i^n)}{(1+\tau_i^x)} \frac{w}{z} \right\} s \left\{ p\gamma \mu^{\mu\gamma} (1-\mu)^{1-\mu\gamma} \left[\frac{w}{(1-\alpha-\beta)} \right]^{(\alpha+\beta)(1-\mu)\gamma+\mu\gamma-1} \right. \\
&\quad \left. \left[\left(\frac{\alpha}{r} \right)^\alpha \left(\frac{\beta}{q} \right)^\beta \right]^{(1-\mu)\gamma} \left(\frac{1}{z} \right)^{\mu\gamma} \frac{(1+\tau_i^n)^{\alpha(1-\mu)\gamma+\beta(1-\mu)\gamma+\mu\gamma-1}}{(1+\tau_i^k)^{\alpha(1-\mu)\gamma} (1+\tau_i^l)^{\beta(1-\mu)\gamma} (1+\tau_i^x)^{\mu\gamma}} \right\}^{\frac{1}{1-\gamma}} \\
&= s_i \left\{ p\gamma \mu^{\mu\gamma+1-\gamma} (1-\mu)^{\gamma-\mu\gamma} \left[\frac{w}{(1-\alpha-\beta)} \right]^{(1-\mu)\gamma(\alpha+\beta-1)} \left[\left(\frac{\alpha}{r} \right)^\alpha \left(\frac{\beta}{q} \right)^\beta \right]^{(1-\mu)\gamma} \right. \\
&\quad \left. \left(\frac{1}{z} \right)^{\mu\gamma+1-\gamma} \frac{(1+\tau_i^n)^{(1-\mu)\gamma(\alpha+\beta-1)}}{(1+\tau_i^k)^{\alpha(1-\mu)\gamma} (1+\tau_i^l)^{\beta(1-\mu)\gamma} (1+\tau_i^x)^{\mu\gamma+1-\gamma}} \right\}^{\frac{1}{1-\gamma}}
\end{aligned}$$

Farm output in equilibrium

$$\begin{aligned}
y_i &= s_i^{1-\gamma} \left[\left(k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta} \right)^{1-\mu} x_i^\mu \right]^\gamma \\
&= s_i^{1-\gamma} \left[\left(\frac{k_i}{n_i} \right)^\alpha \left(\frac{l_i}{n_i} \right)^\beta n_i \right]^{(1-\mu)\gamma} x_i^{\mu\gamma} . \\
&\quad \left\{ s_i \left\{ p\gamma \mu^{\mu\gamma} (1-\mu)^{1-\mu\gamma} \left[\frac{w}{(1-\alpha-\beta)} \right]^{(\alpha+\beta)(1-\mu)\gamma+\mu\gamma-1} \left[\left(\frac{\alpha}{r} \right)^\alpha \left(\frac{\beta}{q} \right)^\beta \right]^{(1-\mu)\gamma} \right. \right. \\
&\quad \left. \left(\frac{1}{z} \right)^{\mu\gamma} \frac{(1+\tau_i^n)^{\alpha(1-\mu)\gamma+\beta(1-\mu)\gamma+\mu\gamma-1}}{(1+\tau_i^k)^{\alpha(1-\mu)\gamma} (1+\tau_i^l)^{\beta(1-\mu)\gamma} (1+\tau_i^x)^{\mu\gamma}} \right\}^{\frac{1}{1-\gamma}} \right\}^{(1-\mu)\gamma} \\
&\quad \left\{ s_i \left\{ p\gamma \mu^{\mu\gamma+1-\gamma} (1-\mu)^{\gamma-\mu\gamma} \left[\frac{w}{(1-\alpha-\beta)} \right]^{(1-\mu)\gamma(\alpha+\beta-1)} \left[\left(\frac{\alpha}{r} \right)^\alpha \left(\frac{\beta}{q} \right)^\beta \right]^{(1-\mu)\gamma} \right. \right. \\
&\quad \left. \left(\frac{1}{z} \right)^{\mu\gamma+1-\gamma} \frac{(1+\tau_i^n)^{(1-\mu)\gamma(\alpha+\beta-1)}}{(1+\tau_i^k)^{\alpha(1-\mu)\gamma} (1+\tau_i^l)^{\beta(1-\mu)\gamma} (1+\tau_i^x)^{\mu\gamma+1-\gamma}} \right\}^{\frac{1}{1-\gamma}} \right\}^{\mu\gamma} \\
y_i &= s_i (p\gamma)^{\frac{\gamma}{1-\gamma}} \mu^{\mu \frac{\gamma}{1-\gamma}} (1-\mu)^{(1-\mu) \frac{\gamma}{1-\gamma}} \left(\frac{1}{z} \right)^{\mu \frac{\gamma}{1-\gamma}} \left(\frac{w}{1-\alpha-\beta} \right)^{(1-\mu)(\alpha+\beta-1) \frac{\gamma}{1-\gamma}} \\
&\quad \left(\frac{\alpha}{r} \right)^{\alpha(1-\mu) \frac{\gamma}{1-\gamma}} \left(\frac{\beta}{q} \right)^{\beta(1-\mu) \frac{\gamma}{1-\gamma}} \frac{(1+\tau_i^n)^{(\alpha+\beta-1)(1-\mu) \frac{\gamma}{1-\gamma}}}{(1+\tau_i^k)^{\alpha(1-\mu) \frac{\gamma}{1-\gamma}} (1+\tau_i^l)^{\beta(1-\mu) \frac{\gamma}{1-\gamma}} (1+\tau_i^x)^{\mu \frac{\gamma}{1-\gamma}}}
\end{aligned}$$

$$y_i = s_i \left\{ (p\gamma) (1-\mu)^{(1-\mu)} \left[\frac{1-\alpha-\beta}{w(1+\tau_i^n)} \right]^{(1-\alpha-\beta)(1-\mu)} \left[\frac{\alpha}{r(1+\tau_i^k)} \right]^{\alpha(1-\mu)} \left[\frac{\beta}{q(1+\tau_i^l)} \right]^{\beta(1-\mu)} \left[\frac{\mu}{z(1+\tau_i^x)} \right]^\mu \right\}^{\frac{\gamma}{1-\gamma}}$$

Physical Productivity

$$\text{TFPQ}_i = s_i^{1-\gamma} = \frac{y_i}{\left[\left(k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta} \right)^{1-\mu} x_i^\mu \right]^\gamma}$$

Revenue Productivity

$$\begin{aligned} \text{TFPR}_i &= \frac{py_i}{\left(k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta} \right)^{1-\mu} x_i^\mu} = \left(\frac{py_i}{k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta}} \right)^{1-\mu} \left(\frac{py_i}{x_i} \right)^\mu \\ &= \frac{(py_i) (py_i)^{-(1-\alpha-\beta)(1-\mu)} (py_i)^{(1-\alpha-\beta)(1-\mu)}}{\left(k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta} \right)^{1-\mu} x_i^\mu} \\ &= \left(\frac{py_i}{k_i} \right)^{\alpha(1-\mu)} \left(\frac{py_i}{l_i} \right)^{\beta(1-\mu)} \left(\frac{py_i}{n_i} \right)^{(1-\alpha-\beta)(1-\mu)} \left(\frac{py_i}{x_i} \right)^\mu \\ &= \left[\left(\frac{py_i}{k_i} \right)^\alpha \left(\frac{py_i}{l_i} \right)^\beta \left(\frac{py_i}{n_i} \right)^{(1-\alpha-\beta)} \right]^{(1-\mu)} \left[\left(\frac{py_i}{x_i} \right) \right]^\mu \end{aligned}$$

$$\begin{aligned}
\text{MRPK}_i &\equiv (1 - \mu)\gamma\alpha\frac{py_i}{k_i} = (1 + \tau_i^k)r \implies \frac{\text{MRPK}_i}{(1 - \mu)\gamma\alpha} \equiv \frac{py_i}{k_i} = \frac{(1 + \tau_i^k)r}{(1 - \mu)\gamma\alpha} \\
\text{MRPL}_i &\equiv (1 - \mu)\gamma\beta\frac{py_i}{l_i} = (1 + \tau_i^l)q \implies \frac{\text{MRPL}_i}{(1 - \mu)\gamma\beta} \equiv \frac{py_i}{l_i} = \frac{(1 + \tau_i^l)q}{(1 - \mu)\gamma\beta} \\
\text{MRPN}_i &\equiv (1 - \mu)\gamma(1 - \alpha - \beta)\frac{py_i}{n_i} = (1 + \tau_i^n)w \implies \\
\frac{\text{MRPN}_i}{(1 - \mu)\gamma(1 - \alpha - \beta)} &\equiv \frac{py_i}{n_i} = \frac{(1 + \tau_i^n)w}{(1 - \mu)\gamma(1 - \alpha - \beta)} \\
\text{MRPX}_i &\equiv \mu\frac{py_i}{x_i} = (1 + \tau_i^x)z \implies \frac{\text{MRPX}_i}{\gamma\mu} \equiv \frac{py_i}{x_i} = \frac{(1 + \tau_i^x)z}{\gamma\mu}
\end{aligned}$$

$$\begin{aligned}
\text{TFPR}_i &= \left\{ \left[\frac{\text{MRPK}_i}{(1 - \mu)\gamma\alpha} \right]^\alpha \left[\frac{\text{MRPL}_i}{(1 - \mu)\gamma\beta} \right]^\beta \left[\frac{\text{MRPN}_i}{(1 - \mu)\gamma(1 - \alpha - \beta)} \right]^{1 - \alpha - \beta} \right\}^{1 - \mu} \left(\frac{\text{MRPX}_i}{\gamma\mu} \right)^\mu \\
&= \left[\left\{ \left[\frac{(1 + \tau_i^k)r}{\gamma(1 - \mu)\alpha} \right]^\alpha \left[\frac{(1 + \tau_i^l)q}{\gamma(1 - \mu)\beta} \right]^\beta \left[\frac{(1 + \tau_i^n)w}{\gamma(1 - \mu)(1 - \alpha - \beta)} \right]^{1 - \alpha - \beta} \right\} \right]^{1 - \mu} \left(\frac{(1 + \tau_i^x)z}{\gamma\mu} \right)^\mu \\
&= \left[(1 + \tau_i^k)^\alpha (1 + \tau_i^l)^\beta (1 + \tau_i^n)^{1 - \alpha - \beta} \right]^{1 - \mu} (1 + \tau_i^x)^\mu \cdot \\
&\quad \left\{ \left[\frac{r}{\gamma(1 - \mu)\alpha} \right]^\alpha \left[\frac{q}{\gamma(1 - \mu)\beta} \right]^\beta \left[\frac{w}{\gamma(1 - \mu)(1 - \alpha - \beta)} \right]^{1 - \alpha - \beta} \right\}^{1 - \mu} \left(\frac{z}{\gamma\mu} \right)^\mu
\end{aligned}$$

Derivation of factor demand in terms of weighted marginal revenue products, individual distortions and aggregate resources:

$$\gamma(1 - \mu)\alpha\frac{py_i}{k_i} = (1 + \tau_i^k)r$$

$$\gamma(1 - \mu)\alpha\frac{py_i}{(1 + \tau_i^k)} = rk_i \quad (\text{A1})$$

$$\gamma(1 - \mu)\alpha \sum \frac{\frac{py_i}{(1 + \tau_i^k)}}{pY} = \frac{r \sum k_i}{pY} \equiv \frac{rK}{pY} \quad (\text{A2})$$

Divide (A1) by (A2) gives demand for capital

$$\begin{aligned}
k_i &= \left[\frac{\frac{py_i}{(1+\tau_i^k)} \frac{1}{pY}}{\sum \frac{py_i}{(1+\tau_i^k)} \frac{1}{pY}} \right] K \\
&= \left[\frac{r}{\underbrace{\sum \frac{py_i}{(1+\tau_i^k)} \frac{1}{pY}}_{\overline{\text{MRPK}}} \frac{\frac{py_i}{(1+\tau_i^k)} \frac{1}{pY}}{r}} \right] K \\
&= \overline{\text{MRPK}} \left[\frac{\frac{py_i}{(1+\tau_i^k)} \frac{1}{pY}}{r} \right] K
\end{aligned}$$

$$\gamma(1-\mu)\beta \frac{py_i}{l_i} = (1+\tau_i^l)q$$

$$\gamma(1-\mu)\beta \frac{py_i}{(1+\tau_i^l)} = ql_i \quad (\text{A3})$$

$$\gamma(1-\mu)\beta \sum \frac{\frac{py_i}{(1+\tau_i^l)}}{pY} = \frac{q \sum l_i}{pY} \equiv \frac{qL}{pY} \quad (\text{A4})$$

Dividing (A3) by (A4) gives demand for land

$$\begin{aligned}
l_i &= \left[\frac{\frac{py_i}{(1+\tau_i^l)} \frac{1}{pY}}{\sum \frac{py_i}{(1+\tau_i^l)} \frac{1}{pY}} \right] L \\
&= \left[\frac{q}{\underbrace{\sum \frac{py_i}{(1+\tau_i^l)} \frac{1}{pY}}_{\overline{\text{MRPL}}} \frac{\frac{py_i}{(1+\tau_i^l)} \frac{1}{pY}}{q}} \right] L \\
&= \overline{\text{MRPL}} \left[\frac{\frac{py_i}{(1+\tau_i^l)} \frac{1}{pY}}{q} \right] L
\end{aligned}$$

$$\gamma(1-\mu)(1-\alpha-\beta)\frac{py_i}{n_i} = (1+\tau_i^n)w$$

$$\gamma(1-\mu)(1-\alpha-\beta)\frac{py_i}{(1+\tau_i^n)} = wn_i \quad (\text{A5})$$

$$\gamma(1-\mu)(1-\alpha-\beta)\sum \frac{\frac{py_i}{(1+\tau_i^n)}}{pY} = \frac{w\sum n_i}{pY} \equiv \frac{wN}{pY} \quad (\text{A6})$$

Divide (A5) by (A6) gives demand for labor

$$\begin{aligned} n_i &= \left[\frac{\frac{py_i}{(1+\tau_i^n)} \frac{1}{pY}}{\sum \frac{py_i}{(1+\tau_i^n)} \frac{1}{pY}} \right] N \\ &= \left[\frac{\frac{w}{\underbrace{\sum \frac{py_i}{(1+\tau_i^n)} \frac{1}{pY}}_{\overline{\text{MRPN}}}} \frac{\frac{py_i}{(1+\tau_i^n)} \frac{1}{pY}}{w}}{1} \right] N \\ &= \overline{\text{MRPN}} \left[\frac{\frac{py_i}{(1+\tau_i^n)} \frac{1}{pY}}{w} \right] N \end{aligned}$$

$$\gamma\mu\frac{py_i}{x_i} = (1+\tau_i^x)z$$

$$\gamma\mu\frac{py_i}{(1+\tau_i^x)} = zx_i \quad (\text{A7})$$

$$\gamma\mu\sum \frac{\frac{py_i}{(1+\tau_i^x)}}{pY} = \frac{z\sum x_i}{pY} \equiv \frac{zX}{pY} \quad (\text{A8})$$

Dividing (A7) by (A8) gives demand for intermediate inputs

$$\begin{aligned}
x_i &= \frac{\left[\frac{\frac{py_i}{(1+\tau_i^x)} \frac{1}{pY}}{\sum \frac{py_i}{(1+\tau_i^x)} \frac{1}{pY}} \right] X}{\left[\frac{\frac{z}{\underbrace{\sum \frac{py_i}{(1+\tau_i^x)} \frac{1}{pY}}_{\overline{\text{MRPX}}}} \frac{\frac{py_i}{(1+\tau_i^x)} \frac{1}{pY}}{z} \right] X} \\
&= \overline{\text{MRPX}} \left[\frac{\frac{py_i}{(1+\tau_i^x)} \frac{1}{pY}}{z} \right] X
\end{aligned}$$

Substituting k_i, l_i, n_i and x_i in terms of (10), (11), (12) and (13) into

$$y_i = s_i^{1-\gamma} \left[\left(k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta} \right)^{1-\mu} x_i^\mu \right]^\gamma \text{ gives}$$

$$\begin{aligned}
y_i &= s_i^{1-\gamma} \left[\left(k_i^\alpha l_i^\beta n_i^{1-\alpha-\beta} \right)^{1-\mu} x_i^\mu \right]^\gamma \\
&= s_i^{1-\gamma} \left[(\overline{\text{MRPK}})^\alpha (\overline{\text{MRPL}})^\beta (\overline{\text{MRPN}})^{1-\alpha-\beta} \right]^{(1-\mu)\gamma} (\overline{\text{MRPX}})^{\mu\gamma} \left(K^\alpha L^\beta N^{1-\alpha-\beta} \right)^{(1-\mu)\gamma} X^{\mu\gamma} \left(\frac{py_i}{pY} \right)^\gamma \\
&\quad \left(\left[\frac{1}{(1+\tau_i^k)r} \right]^\alpha \left[\frac{1}{(1+\tau_i^l)q} \right]^\beta \left[\frac{1}{(1+\tau_i^n)w} \right]^{1-\alpha-\beta} \right)^{(1-\mu)\gamma} \left[\frac{1}{(1+\tau_i^x)z} \right]^{\mu\gamma} \\
y_i^{1-\gamma} &= s_i^{1-\gamma} \left[(\overline{\text{MRPK}})^\alpha (\overline{\text{MRPL}})^\beta (\overline{\text{MRPN}})^{1-\alpha-\beta} \right]^{(1-\mu)\gamma} (\overline{\text{MRPX}})^{\mu\gamma} \left(K^\alpha L^\beta N^{1-\alpha-\beta} \right)^{(1-\mu)\gamma} X^{\mu\gamma} \left(\frac{1}{Y} \right)^\gamma \\
&\quad \left(\left[\frac{1}{(1+\tau_i^k)r} \right]^\alpha \left[\frac{1}{(1+\tau_i^l)q} \right]^\beta \left[\frac{1}{(1+\tau_i^n)w} \right]^{1-\alpha-\beta} \right)^{(1-\mu)\gamma} \left[\frac{1}{(1+\tau_i^x)z} \right]^{\mu\gamma} \\
y_i &= \left\{ s_i \left(\frac{1}{Y} \right)^{\frac{\gamma}{1-\gamma}} \left[(\overline{\text{MRPK}})^\alpha (\overline{\text{MRPL}})^\beta (\overline{\text{MRPN}})^{1-\alpha-\beta} \right]^{\frac{(1-\mu)\gamma}{1-\gamma}} (\overline{\text{MRPX}})^{\frac{\mu\gamma}{1-\gamma}} \left(K^\alpha L^\beta N^{1-\alpha-\beta} \right)^{\frac{(1-\mu)\gamma}{1-\gamma}} \right. \\
&\quad \left. X^{\frac{\mu\gamma}{1-\gamma}} \left(\left[\frac{1}{(1+\tau_i^k)r} \right]^\alpha \left[\frac{1}{(1+\tau_i^l)q} \right]^\beta \left[\frac{1}{(1+\tau_i^n)w} \right]^{1-\alpha-\beta} \right)^{\frac{(1-\mu)\gamma}{1-\gamma}} \left[\frac{1}{(1+\tau_i^x)z} \right]^{\frac{\mu\gamma}{1-\gamma}} \right\}
\end{aligned}$$

Derivation of Aggregate Agricultural Output and TFP:

Summing over all farms, $\sum y_i = Y$

$$Y = \left\{ \left(\frac{1}{Y} \right)^{\frac{\gamma}{1-\gamma}} \left[(\overline{\text{MRPK}})^\alpha (\overline{\text{MRPL}})^\beta (\overline{\text{MRPN}})^{1-\alpha-\beta} \right]^{\frac{(1-\mu)\gamma}{1-\gamma}} (\overline{\text{MRPX}})^{\frac{\mu\gamma}{1-\gamma}} \left(K^\alpha L^\beta N^{1-\alpha-\beta} \right)^{\frac{(1-\mu)\gamma}{1-\gamma}} X^{\frac{\mu\gamma}{1-\gamma}} \sum s_i \left(\left[\frac{1}{(1+\tau_i^k)r} \right]^\alpha \left[\frac{1}{(1+\tau_i^l)q} \right]^\beta \left[\frac{1}{(1+\tau_i^n)w} \right]^{1-\alpha-\beta} \right)^{\frac{(1-\mu)\gamma}{1-\gamma}} \left[\frac{1}{(1+\tau_i^x)z} \right]^{\frac{\mu\gamma}{1-\gamma}} \right\}$$

$$Y^{\frac{1}{1-\gamma}} = \left[(\overline{\text{MRPK}})^\alpha (\overline{\text{MRPL}})^\beta (\overline{\text{MRPN}})^{1-\alpha-\beta} \right]^{\frac{(1-\mu)\gamma}{1-\gamma}} (\overline{\text{MRPX}})^{\frac{\mu\gamma}{1-\gamma}} \left(K^\alpha L^\beta N^{1-\alpha-\beta} \right)^{\frac{(1-\mu)\gamma}{1-\gamma}} X^{\frac{\mu\gamma}{1-\gamma}} \sum s_i \left(\left[\frac{1}{(1+\tau_i^k)r} \right]^\alpha \left[\frac{1}{(1+\tau_i^l)q} \right]^\beta \left[\frac{1}{(1+\tau_i^n)w} \right]^{1-\alpha-\beta} \right)^{\frac{(1-\mu)\gamma}{1-\gamma}} \left[\frac{1}{(1+\tau_i^x)z} \right]^{\frac{\mu\gamma}{1-\gamma}}$$

$$Y = \text{TFP} \left(K^\alpha L^\beta N^{1-\alpha-\beta} \right)^{(1-\mu)\gamma} X^{\mu\gamma} M^{1-\gamma}$$

Therefore, aggregate TFP is

$$\text{TFP} = \left[(\overline{\text{MRPK}})^\alpha (\overline{\text{MRPL}})^\beta (\overline{\text{MRPN}})^{1-\alpha-\beta} \right]^{(1-\mu)\gamma} (\overline{\text{MRPX}})^{\mu\gamma} \left\{ \frac{\sum s_i \left(\left[\frac{1}{(1+\tau_i^k)r} \right]^\alpha \left[\frac{1}{(1+\tau_i^l)q} \right]^\beta \left[\frac{1}{(1+\tau_i^n)w} \right]^{1-\alpha-\beta} \right)^{(1-\mu)\frac{\gamma}{1-\gamma}} \left[\frac{1}{(1+\tau_i^x)z} \right]^{\mu\frac{\gamma}{1-\gamma}}}{M} \right\}^{1-\gamma}$$

which can be simplified to

$$\begin{aligned}
\text{TFP} &= \left[\left(\frac{\overline{\text{MRPK}}}{\gamma(1-\mu)\alpha} \right)^\alpha \left(\frac{\overline{\text{MRPL}}}{\gamma(1-\mu)\beta} \right)^\beta \left(\frac{\overline{\text{MRPN}}}{\gamma(1-\mu)(1-\alpha-\beta)} \right)^{1-\alpha-\beta} \right]^{(1-\mu)\gamma} \left(\frac{\overline{\text{MRPX}}}{\gamma\mu} \right)^{\mu\gamma} \\
&\quad \left\{ [\gamma(1-\mu)\alpha]^\alpha [\gamma(1-\mu)\beta]^\beta [\gamma(1-\mu)(1-\alpha-\beta)]^{1-\alpha-\beta} \right\}^{(1-\mu)\gamma} \gamma\mu^{\mu\gamma} \\
&\quad \left\{ \frac{\sum s_i \left(\left[\frac{1}{(1+\tau_i^k)r} \right]^\alpha \left[\frac{1}{(1+\tau_i^l)q} \right]^\beta \left[\frac{1}{(1+\tau_i^n)w} \right]^{1-\alpha-\beta} \right)^{(1-\mu)\frac{\gamma}{1-\gamma}} \left[\frac{1}{(1+\tau_i^x)z} \right]^\mu \frac{\gamma}{1-\gamma}}{M} \right\}^{1-\gamma} \\
&= \left\{ \left[\left(\frac{\overline{\text{MRPK}}}{\gamma(1-\mu)\alpha} \right)^\alpha \left(\frac{\overline{\text{MRPL}}}{\gamma(1-\mu)\beta} \right)^\beta \left(\frac{\overline{\text{MRPN}}}{\gamma(1-\mu)(1-\alpha-\beta)} \right)^{1-\alpha-\beta} \right]^{(1-\mu)} \left(\frac{\overline{\text{MRPX}}}{\gamma\mu} \right)^\mu \right\}^\gamma \\
&\quad \left\{ \left[\frac{\gamma(1-\mu)\alpha}{r} \right]^\alpha \left[\frac{\gamma(1-\mu)\beta}{q} \right]^\beta \left[\frac{\gamma(1-\mu)(1-\alpha-\beta)}{w} \right]^{1-\alpha-\beta} \right\}^{(1-\mu)\gamma} \left(\frac{\gamma\mu}{z} \right)^{\mu\gamma} \\
&\quad \left\{ \frac{\sum s_i \left(\left[\frac{1}{(1+\tau_i^k)} \right]^\alpha \left[\frac{1}{(1+\tau_i^l)} \right]^\beta \left[\frac{1}{(1+\tau_i^n)} \right]^{1-\alpha-\beta} \right)^{(1-\mu)\frac{\gamma}{1-\gamma}} \left[\frac{1}{(1+\tau_i^x)} \right]^\mu \frac{\gamma}{1-\gamma}}{M} \right\}^{1-\gamma}
\end{aligned}$$

$$\begin{aligned}
\text{TFP} &= \left\{ \left[\left(\frac{\overline{\text{MRPK}}}{\gamma(1-\mu)\alpha} \right)^\alpha \left(\frac{\overline{\text{MRPL}}}{\gamma(1-\mu)\beta} \right)^\beta \left(\frac{\overline{\text{MRPN}}}{\gamma(1-\mu)(1-\alpha-\beta)} \right)^{1-\alpha-\beta} \right]^{(1-\mu)} \left(\frac{\overline{\text{MRPX}}}{\gamma\mu} \right)^\mu \right\}^\gamma \\
&\quad \left\{ \sum \frac{s_i \left\{ \Gamma \left(\left[\frac{1}{r\gamma(1-\mu)\alpha} \right]^\alpha \left[\frac{1}{q\gamma(1-\mu)\beta} \right]^\beta \left[\frac{1}{w\gamma(1-\mu)(1-\alpha-\beta)} \right]^{1-\alpha-\beta} \right)^{(1-\mu)} \left(\frac{1}{z\gamma\mu} \right)^\mu \right\}^{\frac{\gamma}{1-\gamma}}}{M} \right\}^{1-\gamma}
\end{aligned}$$

$$\text{where } \Gamma = \frac{1}{[(1+\tau_i^k)^\alpha (1+\tau_i^l)^\beta (1+\tau_i^n)^{1-\alpha-\beta}]^{(1-\mu)} (1+\tau_i^x)^\mu}$$

$$\overline{\text{TFPR}} = \left[\left\{ \left[\frac{\overline{\text{MRPK}}_i}{\gamma(1-\mu)\alpha} \right]^\alpha \left[\frac{\overline{\text{MRPL}}_i}{\gamma(1-\mu)\beta} \right]^\beta \left[\frac{\overline{\text{MRPN}}_i}{\gamma(1-\mu)(1-\alpha-\beta)} \right]^{1-\alpha-\beta} \right\}^{1-\mu} \left(\frac{\overline{\text{MRPX}}_i}{\gamma\mu} \right)^\mu \right]$$

and

$$\text{TFPR}_i = \left(\frac{1}{\Gamma}\right) \left\{ \left[\frac{r}{\gamma(1-\mu)\alpha} \right]^\alpha \left[\frac{q}{\gamma(1-\mu)\beta} \right]^\beta \left[\frac{w}{\gamma(1-\mu)(1-\alpha-\beta)} \right]^{1-\alpha-\beta} \right\}^{1-\mu} \left(\frac{z}{\gamma\mu} \right)^\mu$$

therefore

$$\frac{\overline{\text{TFPR}}}{\text{TFPR}_i} = \left[\left\{ \left[\frac{\overline{\text{MRPK}}_i}{\overline{\text{MRPK}}} \right]^\alpha \left[\frac{\overline{\text{MRPL}}_i}{\overline{\text{MRPL}}} \right]^\beta \left[\frac{\overline{\text{MRPN}}_i}{\overline{\text{MRPN}}} \right]^{1-\alpha-\beta} \right\} \right]^{1-\mu} \left(\frac{\overline{\text{MRPX}}_i}{\overline{\text{MRPX}}} \right)^\mu$$

$$\begin{aligned} \text{TFP}^{\text{distorted}} &= \overline{\text{TFPR}}^\gamma \left\{ \frac{\sum s_i \frac{1}{\text{TFPR}_i^{\frac{\gamma}{1-\gamma}}}}{M} \right\}^{1-\gamma} \\ &= \left\{ \sum \frac{s_i \left(\frac{\overline{\text{TFPR}}}{\text{TFPR}_i} \right)^{\frac{\gamma}{1-\gamma}}}{M} \right\}^{1-\gamma} \end{aligned}$$

Substituting $s_i^{1-\gamma} = \text{TFPQ}_i$

$$\begin{aligned} \text{TFP}^{\text{distorted}} &= \left\{ \sum \frac{\text{TFPQ}_i^{\frac{1}{1-\gamma}} \left(\frac{\overline{\text{TFPR}}}{\text{TFPR}_i} \right)^{\frac{\gamma}{1-\gamma}}}{M} \right\}^{1-\gamma} \\ &= \left\{ \sum \frac{\left[\text{TFPQ}_i \left(\frac{\overline{\text{TFPR}}}{\text{TFPR}_i} \right)^\gamma \right]^{\frac{1}{1-\gamma}}}{M} \right\}^{1-\gamma} \end{aligned}$$

$$\text{TFP}^{\text{efficient}} = \left\{ \frac{\sum \text{TFPQ}_i^{\frac{1}{1-\gamma}}}{M} \right\}^{1-\gamma}$$

$$\text{TFP}^{\text{gain}} = \left\{ \frac{\left\{ \sum \text{TFPQ}_i^{\frac{1}{1-\gamma}} \right\}^{1-\gamma}}{\left\{ \sum \left[\text{TFPQ}_i \left(\frac{\overline{\text{TFPR}}}{\text{TFPR}_i} \right)^\gamma \right]^{\frac{1}{1-\gamma}} \right\}^{1-\gamma}} - 1 \right\} 100\%$$

TFP gain when *net output* is considered

$$\text{TFPQ}_i = \frac{y_i}{\left[k_i^\alpha l_i^\beta (hn)_i^{1-\alpha-\beta} \right]^\gamma}$$

$$\begin{aligned} \overline{\text{TFPR}} &= \left[\frac{\overline{\text{MRPK}}_i}{\gamma\alpha} \right]^\alpha \left[\frac{\overline{\text{MRPL}}_i}{\gamma\beta} \right]^\beta \left[\frac{\overline{\text{MRPN}}_i}{\gamma(1-\alpha-\beta)} \right]^{1-\alpha-\beta} \\ \text{TFPR}_i &= \left[\frac{r(1+\tau_i^k)}{\gamma\alpha} \right]^\alpha \left[\frac{q(1+\tau_i^l)}{\gamma\beta} \right]^\beta \left[\frac{w(1+\tau_i^n)}{\gamma(1-\alpha-\beta)} \right]^{1-\alpha-\beta} \end{aligned}$$

$$\begin{aligned} \text{TFP}^{\text{distorted}} &= \left\{ \frac{\sum \left[\text{TFPQ}_i \left(\frac{\overline{\text{TFPR}}}{\text{TFPR}_i} \right)^\gamma \right]^{\frac{1}{1-\gamma}}}{M} \right\}^{1-\gamma} \\ \text{TFP}^{\text{efficient}} &= \left\{ \frac{\sum \text{TFPQ}_i^{\frac{1}{1-\gamma}}}{M} \right\}^{1-\gamma} \end{aligned}$$

$$\text{TFP}^{\text{gain}} = \left\{ \frac{\left\{ \sum \text{TFPQ}_i^{\frac{1}{1-\gamma}} \right\}^{1-\gamma}}{\left\{ \sum \left[\text{TFPQ}_i \left(\frac{\overline{\text{TFPR}}}{\text{TFPR}_i} \right)^\gamma \right]^{\frac{1}{1-\gamma}} \right\}^{1-\gamma}} - 1 \right\} 100\%$$

B Appendix for Chapter 3

Table B1: Labor Share by Industry (Source: BEA-US Department of Commerce)

ISIC	Industry Classification	Labor Share ($1-\alpha_s$) in Value Added		
		Min	Max	Mean
15	Manufacture of Food Products and Beverages	0.3888	0.4824	0.4458
16	Manufacture of Tobacco Products	0.3888	0.4824	0.4458
17	Manufacture of Textiles	0.5757	0.8697	0.6838
18	Manufacture of Wearing Apparels	0.7138	0.8252	0.7572
19	Manufacture of Leather and Related Products	0.7138	0.8252	0.7572
20	Manufacture of Wood and Related Products	0.7176	0.8661	0.7731
21	Manufacture of Paper and Paper Products	0.4845	0.6433	0.5642
22	Publishing, Printing and Reproduction of Recorded Media	0.8457	0.8840	0.8720
23	Manufacture of Coke and Refined Petroleum Products	0.0911	0.3061	0.1629
24	Manufacture of Chemicals and Chemical Products	0.3470	0.4770	0.4184
25	Manufacture of Rubber and Plastic Products	0.5262	0.6738	0.6001
26	Manufacture of Other Non-Metallic Mineral Products	0.5545	0.7255	0.6459
27	Manufacture of Basic Metals	0.5460	0.7897	0.6570
28	Manufacture of Fabricated Metal Products	0.6512	0.7116	0.6820
29	Manufacture of Machinery and Equipment	0.6333	0.7532	0.6883
31	Manufacture of Electrical Equipment	0.6274	0.7094	0.6588
32	Manufacture of Radio, Television & Communication Equipment	0.5176	1.0520	0.7367
34	Manufacture of Motor Vehicles, Trailers and Semi-trailers	0.6306	1.7783	0.7861
35	Manufacture of Transport Equipment	0.6829	0.8273	0.7514
36	Manufacture of Furniture	0.6607	0.7575	0.7084

Mean over 1998-2011

Combine all industries into an aggregate

$$\begin{aligned} \min C_s &= \sum P_s Y_s \\ \text{subject to } Y &= \prod_{s=1}^S Y_s^{\theta_s} \end{aligned}$$

$$\mathcal{L} = \sum P_s Y_s + \lambda \left[\bar{Y} - \prod Y_s^{\theta_s} \right]$$

Since $\lambda = P$, $P_s Y_s = \theta_s P Y$. Substituting the first order conditions and solving for the aggregate price gives $P \equiv \prod_{s=1}^S (P_s / \theta_s)^{\theta_s}$.

Combine all firms in an industry

$$\begin{aligned}\max \pi_s &= P_s Y_s - \sum P_{si} Y_{si} \\ \text{where } Y_s &= \sum_{i=1}^{M_s} Y_{si}\end{aligned}$$

Differentiating π_s with respect to Y_{si} and solving for each industry's output gives

$$P_s = P_{si}$$

Each firm's profit is given by

$$\begin{aligned}\pi_{si} &= P_{si} Y_{si} - (1 - \tau_{Nsi}) w N_{si} - (1 + \tau_{Ksi}) r K_{si} \\ &= P_{si} \left[A_{si}^{1-\gamma} (K_{si}^{\alpha_s} N_{si}^{1-\alpha_s})^\gamma \right] - (1 - \tau_{Nsi}) w N_{si} - (1 + \tau_{Ksi}) r K_{si}\end{aligned}$$

$$\frac{\partial \pi_{si}}{\partial K_{si}} = \underbrace{P_{si} \gamma \left[A_{si}^{1-\gamma} (K_{si}^{\alpha_s} N_{si}^{1-\alpha_s})^{\gamma-1} \alpha_s K_{si}^{\alpha_s-1} N_{si}^{1-\alpha_s} \right]}_{\text{MRPK}_{si}} = (1 + \tau_{Ksi}) r$$

$$\text{MRPK}_{si} \equiv \gamma \alpha_s \frac{P_s Y_{si}}{K_{si}} = (1 + \tau_{Ksi}) r$$

Similarly

$$\frac{\partial \pi_{si}}{\partial N_{si}} = \underbrace{P_{si} \gamma \left[A_{si}^{1-\gamma} (K_{si}^{\alpha_s} N_{si}^{1-\alpha_s})^{\gamma-1} (1 - \alpha_s) K_{si}^{\alpha_s} N_{si}^{-\alpha_s} \right]}_{\text{MRPN}_{si}} = (1 - \tau_{Nsi}) w$$

$$\text{MRPN}_{si} \equiv \gamma (1 - \alpha_s) \frac{P_s Y_{si}}{N_{si}} = (1 - \tau_{Nsi}) w$$

Dividing the two first order conditions gives the $\frac{K_{si}}{N_{si}}$ ratio = $\frac{\alpha_s}{1-\alpha_s} \frac{w}{r} \frac{(1-\tau_{Nsi})}{(1+\tau_{Ksi})}$ of each firm from which I find the equilibrium labor and capital demand of each firm.

$$\begin{aligned}\gamma P_{si} \left[A_{si}^{1-\gamma} (K_{si}^{\alpha_s} N_{si}^{1-\alpha_s})^{\gamma-1} (1 - \alpha_s) K_{si}^{\alpha_s} N_{si}^{-\alpha_s} \right] &= (1 - \tau_{Nsi}) w \\ A_{si} \left[\gamma P_{si} \left(\frac{1 - \alpha_s}{w} \right)^{1-\alpha_s \gamma} \left(\frac{\alpha_s}{r} \right)^{\alpha_s \gamma} \right]^{\frac{1}{1-\gamma}} \left[\frac{1}{(1 + \tau_{Ksi})^{\alpha_s \gamma} (1 - \tau_{Nsi})^{1-\alpha_s \gamma}} \right]^{\frac{1}{1-\gamma}} &= N_{si}\end{aligned}$$

Substituting labor input into the capital labor ratio solves the equilibrium demand for capital

$$\begin{aligned}
K_{si} &= \frac{\alpha_s}{1-\alpha_s} \frac{w}{r} \frac{(1-\tau_{Nsi})}{(1+\tau_{Ksi})} A_{si} \left[\gamma P_{si} \left(\frac{1-\alpha_s}{w} \right)^{1-\alpha_s\gamma} \left(\frac{\alpha_s}{r} \right)^{\alpha_s\gamma} \right]^{\frac{1}{1-\gamma}} \left[\frac{1}{(1+\tau_{Ksi})^{\alpha_s\gamma} (1-\tau_{Nsi})^{1-\alpha_s\gamma}} \right]^{\frac{1}{1-\gamma}} \\
&= A_{si} \left[\gamma P_{si} \left(\frac{1-\alpha_s}{w} \right)^{\gamma-\alpha_s\gamma} \left(\frac{\alpha_s}{r} \right)^{1-\gamma+\alpha_s\gamma} \right]^{\frac{1}{1-\gamma}} \left[\frac{1}{(1+\tau_{Ksi})^{1-\gamma+\alpha_s\gamma} (1-\tau_{Nsi})^{\gamma-\alpha_s\gamma}} \right]^{\frac{1}{1-\gamma}}
\end{aligned}$$

Substituting capital and labor inputs into $A_{si}^{1-\gamma} (K_{si}^{\alpha_s} N_{si}^{1-\alpha_s})^\gamma$ gives each firm's output as

$$\begin{aligned}
Y_{si} &= A_{si} \left[(\gamma P_{si})^\gamma \left(\frac{1-\alpha_s}{w} \right)^{\gamma-\alpha_s\gamma} \left(\frac{\alpha_s}{r} \right)^{\alpha_s\gamma} \right]^{\frac{1}{1-\gamma}} \left[\frac{1}{(1+\tau_{Ksi})^{\alpha_s\gamma} (1-\tau_{Nsi})^{\gamma-\alpha_s\gamma}} \right]^{\frac{1}{1-\gamma}} \\
&= A_{si} \left[\gamma P_{si} \left(\frac{1-\alpha_s}{w} \right)^{1-\alpha_s} \left(\frac{\alpha_s}{r} \right)^{\alpha_s} \right]^{\frac{\gamma}{1-\gamma}} \left[\frac{1}{(1+\tau_{Ksi})^{\alpha_s} (1-\tau_{Nsi})^{1-\alpha_s}} \right]^{\frac{\gamma}{1-\gamma}}
\end{aligned}$$

To find the aggregate labor in each sector, I aggregate the demand for labor in a sector by aggregating demands of all firms. Then I combine the aggregate demand for labor with the allocation of total expenditure across sectors.

$$(1-\alpha_s)\gamma \frac{P_s Y_{si}}{N_{si}} = (1-\tau_{Nsi})w \quad (B1)$$

$$\begin{aligned}
(1-\alpha_s)\gamma \sum \frac{P_s Y_{si}}{(1-\tau_{Nsi})} &= w \sum N_{si} \\
(1-\alpha_s)\gamma \sum \frac{P_s Y_{si}}{(1-\tau_{Nsi})} \frac{1}{P_s Y_s} &= \frac{w N_s}{P_s Y_s} \quad (B2)
\end{aligned}$$

Divide (B1) by (B2)

$$\begin{aligned}
N_{si} &= \left[\frac{w}{\underbrace{\sum \frac{P_s Y_{si}}{(1-\tau_{Nsi})} \frac{1}{P_s Y_s}}_{\overline{\text{MRPN}}_x}} \frac{\frac{P_s Y_{si}}{(1-\tau_{Nsi})} \frac{1}{P_s Y_s}}{w} \right] N_s \\
N_{si} &= \frac{\overline{\text{MRPN}}_s}{w} \frac{P_s Y_{si}}{(1-\tau_{Nsi})} \frac{1}{P_s Y_s} N_s
\end{aligned}$$

$$\begin{aligned}
N_s &= (1-\alpha_s)\gamma P_s Y_s \frac{\sum \frac{P_s Y_{si}}{(1-\tau_{Nsi})} \frac{1}{P_s Y_s}}{w} \\
&= \frac{(1-\alpha_s)\gamma P_s Y_s}{\overline{\text{MRPN}}_s}
\end{aligned}$$

Since $\sum N_s = N$ and $\theta_s PY = P_s Y_s$,

$$\begin{aligned}
N &= \sum \frac{(1 - \alpha_s) \gamma P_s Y_s}{\overline{\text{MRPN}}_s} \\
N &= \gamma PY \sum \frac{\theta_{s'} (1 - \alpha_{s'})}{\overline{\text{MRPN}}_{s'}} \\
\frac{N}{\sum \frac{\theta_{s'} (1 - \alpha_{s'})}{\overline{\text{MRPN}}_{s'}}} &= \gamma PY
\end{aligned}$$

Substituting γPY this into N_s

$$\begin{aligned}
N_s &= \frac{(1 - \alpha_s) \theta_s}{\overline{\text{MRPN}}_s} \gamma PY \\
&= N \times \frac{(1 - \alpha_s) \theta_s / \overline{\text{MRPN}}_s}{\sum (1 - \alpha_{s'}) \theta_{s'} / \overline{\text{MRPN}}_{s'}}
\end{aligned}$$

The aggregate capital in each sector is found using the method

$$\begin{aligned}
\alpha_s \gamma \frac{P_s Y_{si}}{K_{si}} &= (1 + \tau_{Ksi}) r \\
\alpha_s \gamma \frac{P_s Y_{si}}{(1 + \tau_{Ksi})} &= r K_{si} \quad (\text{B3}) \\
\alpha_s \gamma \sum \frac{P_s Y_{si}}{(1 + \tau_{Ksi})} &= r \sum K_{si} \\
\alpha_s \gamma \sum \frac{P_s Y_{si}}{(1 + \tau_{Ksi})} \frac{1}{P_s Y_s} &= \frac{r K_s}{P_s Y_s} \quad (\text{B4})
\end{aligned}$$

Divide (B3) by (B4)

$$\begin{aligned}
K_{si} &= \left[\frac{r \frac{P_s Y_{si}}{(1 + \tau_{Ksi})} \frac{1}{P_s Y_s}}{\underbrace{\sum \frac{P_s Y_{si}}{(1 + \tau_{Ksi})} \frac{1}{P_s Y_s}}_{\overline{\text{MRPK}}_s}} \frac{1}{r} \right] K_s \\
K_{si} &= \overline{\text{MRPK}}_s \frac{P_s Y_{si}}{(1 + \tau_{Ksi})} \frac{1}{P_s Y_s} K_s
\end{aligned}$$

$$\begin{aligned}
\frac{r K_s}{P_s Y_s} &= \alpha_s \gamma \sum \frac{P_s Y_{si}}{(1 + \tau_{Ksi})} \frac{1}{P_s Y_s} \\
K_s &= \frac{\alpha_s \gamma P_s Y_s}{\overline{\text{MRPK}}_s}
\end{aligned}$$

Since $\sum K_s = K$ and $\theta_s PY = P_s Y_s$

$$\begin{aligned}
K &= \sum \frac{\alpha_s \gamma P_s Y_s}{\overline{\text{MRPK}}_s} \\
K &= \gamma PY \sum \frac{\theta_{s'} \alpha_{s'}}{\overline{\text{MRPK}}_{s'}} \\
\frac{K}{\sum \frac{\theta_{s'} \alpha_{s'}}{\overline{\text{MRPK}}_{s'}}} &= \gamma PY
\end{aligned}$$

,
Substituting γPY this into K_s

$$\begin{aligned}
K_s &= \frac{\alpha_s \theta_s}{\overline{\text{MRPK}}_s} \gamma PY \\
&= K \times \frac{\alpha_s \theta_s / \overline{\text{MRPK}}_s}{\sum \alpha_{s'} \theta_{s'} / \overline{\text{MRPK}}_{s'}}
\end{aligned}$$

Physical Productivity

$$\text{TFPQ}_{si} = A_{si}^{1-\gamma} = \frac{Y_{si}}{(K_{si}^{\alpha_s} N_{si}^{1-\alpha_s})^\gamma}$$

Revenue Productivity

$$\begin{aligned}
\text{TFPR}_{si} &= \frac{P_{si} Y_{si}}{K_{si}^{\alpha_s} N_{si}^{1-\alpha_s}} \\
&= \left(\frac{P_{si} Y_{si}}{K_{si}^{\alpha_s}} \right)^{\alpha_s} \left(\frac{P_{si} Y_{si}}{N_{si}^{1-\alpha_s}} \right)^{1-\alpha_s}
\end{aligned}$$

(1) Derivation of TFPR_{si}

$$\begin{aligned}
\frac{\text{MRPK}_{si}}{\alpha_s \gamma} &\equiv \frac{P_s Y_{si}}{K_{si}} = (1 + \tau_{Ksi}) \frac{r}{\alpha_s \gamma} \\
\frac{\text{MRPN}_{si}}{(1 - \alpha_s)} &\equiv \frac{P_s Y_{si}}{L_{si}} = (1 + \tau_{Nsi}) \frac{w}{(1 - \alpha_s) \gamma}
\end{aligned}$$

$$\begin{aligned}
\text{TFPR}_{si} &= \left(\frac{\text{MRPK}_{si}}{\alpha_s \gamma} \right)^{\alpha_s} \left(\frac{\text{MRPN}_{si}}{(1 - \alpha_s) \gamma} \right)^{1-\alpha_s} \\
&= \left[(1 + \tau_{Ksi}) \frac{r}{\alpha_s \gamma} \right]^{\alpha_s} \left[(1 + \tau_{Nsi}) \frac{w}{(1 - \alpha_s) \gamma} \right]^{1-\alpha_s} \\
&= (1 + \tau_{Ksi})^{\alpha_s} (1 + \tau_{Nsi})^{1-\alpha_s} \left(\frac{r}{\alpha_s \gamma} \right)^{\alpha_s} \left(\frac{w}{(1 - \alpha_s) \gamma} \right)^{1-\alpha_s}
\end{aligned}$$

$$\begin{aligned}
\overline{\text{TFPR}}_s &= \left(\frac{\overline{\text{MRPK}}_{si}}{\alpha_s \gamma} \right)^{\alpha_s} \left(\frac{\overline{\text{MRPN}}_{si}}{(1 - \alpha_s) \gamma} \right)^{1 - \alpha_s} \\
&\quad \left(\frac{1}{\alpha_s \gamma} \frac{r}{\sum \frac{P_s Y_{si}}{(1 + \tau_{Ksi})} \frac{1}{P_s Y_s}} \right)^{\alpha_s} \left(\frac{1}{(1 - \alpha_s) \gamma} \frac{w}{\sum \frac{P_s Y_{si}}{(1 + \tau_{Nsi})} \frac{1}{P_s Y_s}} \right)^{(1 - \alpha_s)} \\
&= \left(\frac{r}{\gamma \alpha_s} \right)^{\alpha_s} \left(\frac{w}{\gamma (1 - \alpha_s)} \right)^{(1 - \alpha_s)} \frac{1}{\left[\sum \frac{P_s Y_{si}}{(1 + \tau_{Ksi})} \frac{1}{P_s Y_s} \right]^{\alpha_s} \left[\sum \frac{P_s Y_{si}}{(1 + \tau_{Nsi})} \frac{1}{P_s Y_s} \right]^{(1 - \alpha_s)}}
\end{aligned}$$

The ratio of $\overline{\text{TFPR}}_s$ to TFPR_{si} gives the following:

$$\frac{\overline{\text{TFPR}}_s}{\text{TFPR}_{si}} = \frac{(1 + \tau_{Ksi})^{-\alpha_s} (1 + \tau_{Nsi})^{-(1 - \alpha_s)}}{\left[\sum \frac{P_s Y_{si}}{(1 + \tau_{Ksi})} \frac{1}{P_s Y_s} \right]^{\alpha_s} \left[\sum \frac{P_s Y_{si}}{(1 + \tau_{Nsi})} \frac{1}{P_s Y_s} \right]^{(1 - \alpha_s)}}$$

B.1 Derivation of Sectoral Output and TFP

$$\begin{aligned}
Y_{si} &= A_{si}^{1 - \gamma} \left(\overline{\text{MRPK}}_s \frac{\frac{P_s Y_{si}}{(1 + \tau_{Ksi})} \frac{1}{P_s Y_s}}{r} K_s \right)^{\alpha_s \gamma} \left(\overline{\text{MRPN}}_s \frac{\frac{P_s Y_{si}}{(1 + \tau_{Nsi})} \frac{1}{P_s Y_s}}{w} N_s \right)^{(1 - \alpha_s) \gamma} \\
&= A_{si}^{1 - \gamma} K_s^{\alpha_s \gamma} N_s^{(1 - \alpha_s) \gamma} (\overline{\text{MRPK}}_s)^{\alpha_s \gamma} (\overline{\text{MRPN}}_s)^{(1 - \alpha_s) \gamma} \left(\frac{P_s Y_{si}}{P_s Y_s} \right)^{\gamma} \left[\frac{1}{(1 + \tau_{Ksi})} \right]^{\alpha_s \gamma} \left[\frac{1}{(1 + \tau_{Nsi})} \right]^{(1 - \alpha_s) \gamma} \\
Y_{si}^{1 - \gamma} &= A_{si}^{1 - \gamma} K_s^{\alpha_s \gamma} N_s^{(1 - \alpha_s) \gamma} (\overline{\text{MRPK}}_s)^{\alpha_s \gamma} (\overline{\text{MRPN}}_s)^{(1 - \alpha_s) \gamma} \left(\frac{1}{Y_s} \right)^{\gamma} \left[\frac{1}{(1 + \tau_{Ksi})} \right]^{\alpha_s \gamma} \left[\frac{1}{(1 + \tau_{Nsi})} \right]^{(1 - \alpha_s) \gamma} \\
Y_{si} &= A_{si} K_s^{\frac{\alpha_s \gamma}{1 - \gamma}} N_s^{\frac{(1 - \alpha_s) \gamma}{1 - \gamma}} (\overline{\text{MRPK}}_s)^{\frac{\alpha_s \gamma}{1 - \gamma}} (\overline{\text{MRPN}}_s)^{\frac{(1 - \alpha_s) \gamma}{1 - \gamma}} \left(\frac{1}{Y_s} \right)^{\frac{\gamma}{1 - \gamma}} \left[\frac{1}{(1 + \tau_{Ksi})} \right]^{\frac{\alpha_s \gamma}{1 - \gamma}} \left[\frac{1}{(1 + \tau_{Nsi})} \right]^{\frac{(1 - \alpha_s) \gamma}{1 - \gamma}} \\
Y_s &= \sum_{i=1}^{M_s} Y_{si} \\
Y_s &= \left\{ \left(\frac{1}{Y_s} \right)^{\frac{\gamma}{1 - \gamma}} K_s^{\frac{\alpha_s \gamma}{1 - \gamma}} N_s^{\frac{(1 - \alpha_s) \gamma}{1 - \gamma}} (\overline{\text{MRPK}}_s)^{\frac{\alpha_s \gamma}{1 - \gamma}} (\overline{\text{MRPN}}_s)^{\frac{(1 - \alpha_s) \gamma}{1 - \gamma}} \right. \\
&\quad \left. \sum A_{si} \left[\frac{1}{r (1 + \tau_{Ksi})} \right]^{\frac{\alpha_s \gamma}{1 - \gamma}} \left[\frac{1}{w (1 + \tau_{Nsi})} \right]^{\frac{(1 - \alpha_s) \gamma}{1 - \gamma}} \right\} \\
&= \left\{ K_s^{\alpha_s \gamma} N_s^{(1 - \alpha_s) \gamma} M_s^{1 - \gamma} \right. \\
&\quad \left. \cdot (\overline{\text{MRPK}}_s)^{\alpha_s \gamma} (\overline{\text{MRPN}}_s)^{(1 - \alpha_s) \gamma} \left\{ \frac{\sum A_{si} \left[\frac{1}{r (1 + \tau_{Ksi})} \right]^{\frac{\alpha_s \gamma}{1 - \gamma}} \left[\frac{1}{w (1 + \tau_{Nsi})} \right]^{\frac{(1 - \alpha_s) \gamma}{1 - \gamma}}}{M_s} \right\}^{1 - \gamma} \right\} \\
&\quad \underbrace{\hspace{15em}}_{\text{TFP}_s}
\end{aligned}$$

$$\begin{aligned}
\text{TFP}_s &= (\overline{\text{MRPK}}_s)^{\alpha_s \gamma} (\overline{\text{MRPN}}_s)^{(1-\alpha_s)\gamma} \left\{ \frac{\sum A_{si} \left[\frac{1}{r(1+\tau_{Ksi})} \right]^{\frac{\alpha_s \gamma}{1-\gamma}} \left[\frac{1}{w(1+\tau_{Nsi})} \right]^{\frac{(1-\alpha_s)\gamma}{1-\gamma}}}{M_s} \right\}^{1-\gamma} \\
&= (\overline{\text{MRPK}}_s)^{\alpha_s \gamma} (\overline{\text{MRPN}}_s)^{(1-\alpha_s)\gamma} \left\{ \frac{\sum A_{si} \left[\frac{1}{r(1+\tau_{Ksi})} \right]^{\frac{\alpha_s \gamma}{1-\gamma}} \left[\frac{1}{w(1+\tau_{Nsi})} \right]^{\frac{(1-\alpha_s)\gamma}{1-\gamma}}}{M_s} \right\}^{1-\gamma} \\
\text{TFPR}_{si} &= \left(\frac{r}{\gamma \alpha_s} \right)^{\alpha_s} \left(\frac{w}{\gamma(1-\alpha_s)} \right)^{(1-\alpha_s)} \frac{1}{\left[\sum \frac{P_s Y_{si}}{(1+\tau_{Ksi})} \frac{1}{P_s Y_s} \right]^{\alpha_s} \left[\sum \frac{P_s Y_{si}}{(1+\tau_{Nsi})} \frac{1}{P_s Y_s} \right]^{(1-\alpha_s)}} \\
\text{TFP}_s &= \left[\left(\frac{\overline{\text{MRPK}}_s}{\alpha_s \gamma} \right)^{\alpha_s} \left(\frac{\overline{\text{MRPN}}_s}{(1-\alpha_s)\gamma} \right)^{(1-\alpha_s)} \right]^\gamma \left\{ \frac{\sum A_{si} \left[\frac{\alpha_s \gamma}{r(1+\tau_{Ksi})} \right]^{\frac{\alpha_s \gamma}{1-\gamma}} \left[\frac{(1-\alpha_s)\gamma}{w(1+\tau_{Nsi})} \right]^{\frac{(1-\alpha_s)\gamma}{1-\gamma}}}{M} \right\}^{1-\gamma} \\
\text{TFP}_s &= \left[\left(\frac{\overline{\text{MRPK}}_s}{\alpha_s \gamma} \right)^{\alpha_s} \left(\frac{\overline{\text{MRPN}}_s}{(1-\alpha_s)\gamma} \right)^{(1-\alpha_s)} \right]^\gamma \left\{ \frac{\sum A_{si} \left(\left[\frac{\alpha_s \gamma}{r(1+\tau_{Ksi})} \right]^{\alpha_s} \left[\frac{(1-\alpha_s)\gamma}{w(1+\tau_{Nsi})} \right]^{1-\alpha_s} \right)^{\frac{\gamma}{1-\gamma}}}{M} \right\}^{1-\gamma} \\
&= \overline{\text{TFPR}}_s^\gamma \left\{ \frac{\sum \text{TFPQ}_{si}^{\frac{1}{1-\gamma}} \left(\frac{1}{\overline{\text{TFPR}}_{si}} \right)^{\frac{\gamma}{1-\gamma}}}{M} \right\}^{1-\gamma} \\
&= \left\{ \frac{\sum \left[\text{TFPQ}_{si} \left(\frac{\overline{\text{TFPR}}_s}{\overline{\text{TFPR}}_{si}} \right)^\gamma \right]^{\frac{1}{1-\gamma}}}{M} \right\}^{1-\gamma} \\
Y &= \prod_{s=1}^S Y_s^{\theta_s} = \prod_{s=1}^S \left(\text{TFP}_s \times K_s^{\alpha_s \gamma} \times N_s^{(1-\alpha_s)\gamma} M_s^{1-\gamma} \right)^{\theta_s} \\
Y &= \prod_{s=1}^S \left\{ \left(K_s^{\alpha_s} N_s^{1-\alpha_s} \right)^\gamma M_s^{1-\gamma} \left\{ \frac{\sum \left[\text{TFPQ}_{si} \left(\frac{\overline{\text{TFPR}}_s}{\overline{\text{TFPR}}_{si}} \right)^\gamma \right]^{\frac{1}{1-\gamma}}}{M} \right\}^{1-\gamma} \right\}^{\theta_s}
\end{aligned}$$

C Appendix for Chapter 4

Each farm's profit is given by

$$\begin{aligned}\pi_{a,i} &= y_{a,i} - (1 + \tau_{a,i}^l)ql_i - (1 + \tau_{a,i}^k)rk_{a,i} \\ &= (A_a s_{a,i})^{1-\lambda} (k_{a,i}^\alpha l_i^{1-\alpha})^\lambda - (1 + \tau_{a,i}^l)ql_i - (1 + \tau_{a,i}^k)rk_{a,i}\end{aligned}$$

$$\begin{aligned}\frac{\partial \pi_{a,i}}{\partial k_{a,i}} &= \underbrace{\lambda \left[(A_a s_{a,i})^{1-\lambda} (k_{a,i}^\alpha l_i^{1-\alpha})^{\lambda-1} \alpha k_{a,i}^{\alpha-1} l_i^{1-\alpha} \right]}_{\text{MRPK}_{a,i}} = (1 + \tau_{a,i}^k)r \\ \text{MRPK}_{a,i} &\equiv \alpha \lambda \frac{y_{a,i}}{k_{a,i}} = (1 + \tau_{a,i}^k)r.\end{aligned}$$

Similarly

$$\begin{aligned}\frac{\partial \pi_{a,i}}{\partial l_i} &= \underbrace{\lambda \left[(A_a s_{a,i})^{1-\lambda} (k_{a,i}^\alpha l_i^{1-\alpha})^{\lambda-1} (1 - \alpha) k_{a,i}^\alpha l_i^{-\alpha} \right]}_{\text{MRPL}_{a,i}} = (1 + \tau_{a,i}^l)q \\ \text{MRPL}_{a,i} &\equiv \lambda(1 - \alpha) \frac{y_{a,i}}{l_i} = (1 + \tau_{a,i}^l)q.\end{aligned}$$

$\text{MRPK}_{a,i}$ and $\text{MRPL}_{a,i}$ are farm-level marginal revenue product of capital and land respectively.

Dividing the two first order conditions gives the capital-land ratio of $\frac{k_{a,i}}{l_i} = \frac{\alpha}{1-\alpha} \frac{q}{r} \frac{(1+\tau_{a,i}^l)}{(1+\tau_{a,i}^k)}$ of each

farm from which I find the equilibrium land and capital demand of each farm.

$$\begin{aligned}
(1 + \tau_{a,i}^l)q &= \lambda \left[(A_a s_{a,i})^{1-\lambda} (k_{a,i}^\alpha l_i^{1-\alpha})^{\lambda-1} (1-\alpha) k_{a,i}^\alpha l_i^{-\alpha} \right] \\
(1 + \tau_{a,i}^l) q l_i^{1-\lambda} &= \lambda (1-\alpha) \left[(A_a s_{a,i})^{1-\lambda} \left(\frac{\alpha}{r} \frac{q}{1-\alpha} \frac{(1 + \tau_{a,i}^l)}{(1 + \tau_{a,i}^k)} \right)^{\alpha\lambda} \right] \\
l_i &= A_a s_{a,i} \left[\lambda \left(\frac{1-\alpha}{q} \right)^{1-\alpha\lambda} \left(\frac{\alpha}{r} \right)^{\alpha\lambda} \right]^{\frac{1}{1-\lambda}} \left[\frac{1}{(1 + \tau_{a,i}^k)^{\alpha\lambda} (1 + \tau_{a,i}^l)^{1-\alpha\lambda}} \right]^{\frac{1}{1-\lambda}}
\end{aligned}$$

Substituting land input into the capital-land ratio solves the equilibrium demand for capital

$$\begin{aligned}
k_{a,i} &= \frac{\alpha}{1-\alpha} \frac{q}{r} \frac{(1 + \tau_{a,i}^l)}{(1 + \tau_{a,i}^k)} A_a s_{a,i} \left[\lambda \left(\frac{1-\alpha}{q} \right)^{1-\alpha\lambda} \left(\frac{\alpha}{r} \right)^{\alpha\lambda} \right]^{\frac{1}{1-\lambda}} \left[\frac{1}{(1 + \tau_{a,i}^k)^{\alpha\lambda} (1 + \tau_{a,i}^l)^{1-\alpha\lambda}} \right]^{\frac{1}{1-\lambda}} \\
&= A_a s_{a,i} \left[\lambda \left(\frac{1-\alpha}{q} \right)^{\lambda-\alpha\lambda} \left(\frac{\alpha}{r} \right)^{1-\lambda+\alpha\lambda} \right]^{\frac{1}{1-\lambda}} \left[\frac{1}{(1 + \tau_{a,i}^k)^{1-\lambda+\alpha\lambda} (1 + \tau_{a,i}^l)^{\lambda-\alpha\lambda}} \right]^{\frac{1}{1-\lambda}}
\end{aligned}$$

Substituting capital and land inputs into $(A_a s_{a,i})^{1-\lambda} (k_{a,i}^\alpha l_i^{1-\alpha})^\lambda$ gives each farm's output as

$$y_{a,i} = A_a s_{a,i} \left[\lambda \left(\frac{1-\alpha}{q} \right)^{1-\alpha} \left(\frac{\alpha}{r} \right)^\alpha \right]^{\frac{\lambda}{1-\lambda}} \left[\frac{1}{(1 + \tau_{a,i}^k)^\alpha (1 + \tau_{a,i}^l)^{1-\alpha}} \right]^{\frac{\lambda}{1-\lambda}}$$

Therefore, farm-level wedges on capital and land are

$$\varphi_{a,i} = \frac{1}{(1 + \tau_{a,i}^k)^\alpha (1 + \tau_{a,i}^l)^{1-\alpha}}.$$

The same derivation can be used to find firm-level wedges on capital and labor to be

$$\varphi_{m,j} = \frac{1}{(1 + \tau_{m,j}^k)^\beta (1 + \tau_{m,j}^n)^{1-\beta}}.$$